Written HW3 – MATH 2502 Spring 2021

Due by 2 September for timely completion credit

In class, we used basic algebra to derive the formula for the inverse hyperbolic sine: $\operatorname{asinh}(x) = \ln(x + \sqrt{x^2+1})$. In this homework, you will carry out the derivation of the logarithmic form of $\operatorname{atanh}(x)$ (valid for $-1 < x < 1$). Recall that

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}. \tag{1}$$

(1) Write $w = \tanh(x)$ and multiply (1) by the denominator.

(2) Write $z = e^x$ and multiply both sides by z.

(3) Solve the resulting quadratic equation for the variable z (*note: you will get w in your answer!*).

(4) Write the inverse you discovered by filling in the following blank:

$$\operatorname{atanh}(x) = \_\_\_\_\_.$$