Written HW25 – MATH 2502 Spring 2021 Due by 19 November for timely completion credit

1. Start with the geometric series $\frac{1}{1-r} = \sum_{k=0}^{\infty} r^k$, replace r with $-t^2$, and then integrate to find a power series for the arctan function:

$$\arctan(x) = \int_0^x \frac{1}{1+t^2} \mathrm{d}t.$$

2. Start with the power series $e^t = \sum_{k=0}^{\infty} \frac{t^k}{k!}$, replace t with $-t^2$, and then integrate to find a power series for the error function erf:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t.$$

3. Start with the power series $\sin(t) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!}$, replace t with t^2 , and then integrate to find a power series for the Fresnel S function

$$S(x) = \int_0^x \sin(t^2) \mathrm{d}t.$$

4. As was done in class, provide a plot for each of the answers (separate plot for each) containing the "actual function" (recall how to input an "integral" into desmos by typing "int") as well as the series approximation to it up to four terms.