Written HW21 – MATH 2502 Spring 2021

Due by 3 November for timely completion credit

Find the interval of convergence and the radius of convergence for the following series. Full details in your calculations should be included, including evaluation (and justification of convergence or divergence) at any endpoints that are relevant.

1. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k (k+1)(k+3) x^k}{(k+2)(k+4)}$

2. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$

3. $\displaystyle\sum_{k=0}^{\infty} \frac{(k^3+1) x^k}{(k+2)(k+4)}$

4. The Bessel function (of the first kind) of order 1 defined by the power series
$$J_1(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+1)! 2^{2k+1}} x^{2k+1}$$