Written HW14 – MATH 2502 Fall 2021

Due by 4 October for timely completion credit

The Laplace transform of a function f is a function, named $\mathscr{L}\{f\}$, given by the following formula:

$$\mathscr{L}\{f\}(x) = \int_0^\infty e^{-xt} f(t)\mathrm{d}t.$$

Laplace transforms appear often in mathematical applications due to their use in the theory of differential equations to solve problems, and they make an appearance in proabability theory in relation to moment-generating functions.

1. Find the antiderivative $\int e^{-xt}\mathrm{d}t$. Be careful to note what the variable of integration is.
2. If $f(t) = 1$, then calculate $\mathscr{L}\{f\}(x)$. Your answer should have "x" in it.
3. If $g(t) = t$, then use integration by parts to calculate $\mathscr{L}\{g\}(x)$. Your answer should have "x" in it.
4. If $h(t) = t^2$, then use integration by parts twice to calculate $\mathscr{L}\{h\}(x)$.
5. Do you see the pattern? What do you think the Laplace transform of t^3 will be? What about the transform of t^n for $n = 4, 5, 6, \ldots$?