

Written HW5 – MATH 2501 Fall 2020

Due by 1 September for timely completion credit

A function f is called left-continuous at a provided that the following three properties hold:

- i.) $f(a)$ exists,
- ii.) $\lim_{x \rightarrow a^-} f(x)$ exists, and
- iii.) $\lim_{x \rightarrow a^-} f(x) = f(a)$.

A function f is called right-continuous at a provided that the same three properties hold, but with the limits being from the right (i.e. $x \rightarrow a^+$). A function f is called continuous at a provided it is simultaneously left-continuous and right-continuous at a . A function f is called continuous on the interval $[a, b]$ if it is continuous at each point of the interval.

1. Find the number k that makes the given function f is continuous at $x = 3$:

$$f(x) = \begin{cases} 5x + k, & x < 3 \\ x^2, & x \geq 3. \end{cases}$$

2. Sketch a picture of a function f defined on the whole interval $[-3, 3]$ which is continuous at $x = 2$, is left-continuous but not right-continuous at $x = 1$, is right-continuous but not left-continuous at $x = -1$, and is neither left- nor right-continuous at $x = -2$.