

Written HW19 – MATH 2501 Fall 2021

**Due by 10 November for timely completion credit**

In this problem, you will find area under the curve “the long way” using the limit of Riemann sums. For each of these problems, the function involved is  $f(x) = 16 - x^2$  and the interval is  $[a, b] = [0, 4]$ .

- (1) Approximate the area under the curve using a left-endpoints Riemann sum with  $n = 3$  rectangles.
- (2) Approximate the area under the curve using a right-endpoints Riemann sum with  $n = 3$  rectangles.
- (3) Compute the actual area under the curve by working out  $\int_0^4 f(x)dx = \lim_{n \rightarrow \infty} \sum_{k=1}^{\infty} f(x_k)\Delta x$ , as was done numerous times in class.
- (4) Explain whether your approximations in (1) and (2) were overestimates or underestimates of the actual area under the curve.