

Written HW11 – MATH 2501 Fall 2021

Due by 27 September for timely completion credit

Full credit on these problems require a fully symbolic derivation of the solutions.

1. The Stefan–Boltzmann law is an equation that relates the power radiated from a black body (e.g. a black hole) in terms of its temperature. It is given by

$$J = \sigma T^4,$$

where J represents the total energy radiated per unit surface area of the black body per unit time, $\sigma \approx 5.670$ (in SI units) is a constant called the Stefan-Boltzmann constant, and T represents temperature (measured in Kelvin, K).

If the temperature is decreasing at a rate of $3 \frac{K}{hr}$, then how is the total energy radiated per unit surface area per unit time changing when the temperature is $T = 100K$?

2. In chemistry, “ pH ” is used to quantify the acidity of a solution. It is defined by the equation

$$pH = -\frac{1}{\ln(10)} \ln(a),$$

where a denotes the hydrogen ion activity measured in the solution.

If the pH of the solution is increasing at a rate of $\frac{0.003}{min}$, then what is the rate of change of hydrogen ion activity when $pH = 3$? (*note: it appears that both pH and “hydrogen ion activity” are “unitless” quantities – this happens sometimes!*)

3. Joule’s law says

$$P = I^2 R,$$

where P is power (in watts), I is the current (in amps), and R is the resistance (in ohms, Ω).

If the current is increasing at a rate of $2 \frac{amps}{sec}$ and power is decreasing at a rate of $3 \frac{watts}{sec}$, then how is resistance changing when the current is 12 amps and the power is 15 watts?