Theorem 15: If $I_1, I_2, I_3, \ldots$ is a sequence of closed intervals such that for each positive integer n, $I_{n+1} \subseteq I_n$ then there is a point p such that is n is any positive integer, then p is in I_n. In other words, there is a point p which is in all closed intervals of the sequence $I_1, I_2, I_3, \ldots$

Proof: As $I_1, I_2, I_3, \ldots$ is a sequence of closed intervals, let $I_1 = [a, b]$, $I_2 = [a_2, b_2]$,… $I_n = [a_n, b_n]$. As all $I_{n+1} \subseteq I_n$, all $[a_{n+1}, b_{n+1}] \subseteq [a_n, b_n]$. Thus, for all positive integers n, $a_n \leq a_{n+1} < b_1$, so the sequence $a_1, a_2, a_3, \ldots$ is non-decreasing and has the point b_1 to the right of every point in the sequence. By Theorem 10, we can say that $a_1, a_2, a_3, \ldots$ converges to some point c. Similarly, $a_1 < b_{n+1} \leq b_n$, so $b_1, b_2, b_3, \ldots$ converges to some point d. Now, as $a_1, a_2, a_3, \ldots$ converges to c and $b_1, b_2, b_3, \ldots$ converges to d, the intersection of all closed intervals of $I_1, I_2, I_3, \ldots$ is $[c, d]$ or $\{c\}$ when $c = d$. If we take a point p such that $p \in [c, d]$, then $p \in I_n$ for all n as $[c, d]$ is a subset of all I_n. Therefore, there must be a point p such that p is in all of the closed intervals of the sequence. ■