Theorem 14: Suppose f and g are functions having domain M and each is continuous at the point p in M. Suppose that h is a function with domain M such that $f(p) = h(p) = g(p)$ and for each number x in M, $f(x) \leq h(x) \leq g(x)$. Prove h is continuous at p.

Proof. Since h is a function with domain M and $p \in M$, then $(p, h(p))$ is a point on h.
Let there exists two horizontal lines containing $(p, h(p))$ called J and K, $J < K$. By definition of continuity, there exists two vertical lines j_f and k_f such that all points $t \in$ domain f that lies between these vertical lines, $(t, f(t))$ is in the rectangle determined by J, K, k_f, j_f. This can be repeated for $(t, g(t))$, which lies in the rectangle determined by J, K, k_g, j_g. Let k_h be the right most of k_f and k_g, and let j_h be the left most of j_f and j_g. Since $f(t) \leq h(t) \leq g(t)$ and $g(t)$ is greater than or equal to $h(t)$ and because $g(t)$ lies between J and K, then $(t, h(t))$ must lie between J and K. Also, $h(t)$ is greater than or equal to $f(t)$ and $f(t)$ lies between J and K, so $h(t)$ must lie between J and K. Now, since $f(p) = h(p) = g(p)$, $(p, h(p))$ can be bounded by vertical lines k_h and j_h. Thus, $(p, h(p))$ is in the rectangle bounded by J, K, j_h, k_h. Therefore, h is continuous at p. □