Theorem 12

Proof:

Assume f is a function and $x_1, x_2, x_3, \ldots$ is a sequence of points in the domain of f converging to the number x in the domain of f, and f is continuous at (x, f(x)).

Since f is continuous at (x, f(x)), if V = (c, d) is any open interval containing f(x) then there is an open interval T containing x such that if t Ɛ T and t is in the domain of f, then f(t) Ɛ V.

Also, since $x_1, x_2, x_3, \ldots$ converges to x and T = (a, b) is an open interval such that x Ɛ T there exists N Ɛ $Ƶ^+$ such that ∀ n ≥ N then x_n Ɛ T.

Thus, we have V = (c, d) such that f(x) Ɛ V and there exists n, N Ɛ $Ƶ^+$ such that ∀ n ≥ N $f(x_n), f(x_{n+1}), f(x_{n+2}), \ldots$ Ɛ V.

Therefore, $f(x_1), f(x_2), f(x_3), \ldots$ converges to f(x).