

Theorem 10: (CA) If p_1, p_2, p_3, \dots is a non-decreasing sequence and there is a point, x , to the right of each point of the sequence, then the sequence converges to some point.

Proof: By the Completeness Axiom, p_1, p_2, p_3, \dots has either a right-most point or a first point to the right.

Case 1: The sequence has a right most point.

Let $p \in \{p_1, p_2, p_3, \dots\}$ be the right-most point. Let S be an open interval (a, b) and $p \in S$. Thus, $\exists N$ such that $p_n = p$. Since the sequence is non-decreasing, $\forall n > N, p_n = p \in S$.

Case 2: The sequence has a first point to the right.

Let p be the first point to the right of $\{p_1, p_2, p_3, \dots\}$. Let (a, b) be any open interval containing p . We know $a < p$, so $\exists p_w \in \{p_1, p_2, p_3, \dots\}$ such that $p_w \in (a, p)$. Thus, $a < p_w < p$ and for any $t > w, p_t \geq p_w$. Further, $p_w \leq p_{w+1} \leq p_{w+2} \leq \dots \leq p \leq b$, so $\forall n > w, p_n \in (a, b)$. Therefore, the sequence converges to some point.