Math 4590

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Theorem 7
If p is a point, there is a sequence of open intervals $S_1, S_2, S_3, ...$ each containing p such that for each positive integer n, $S_{n+1} \subseteq S_n$, and p is the only point that is in every open interval in the sequence.

Proof.
Let $S_n = (p - \frac{1}{n}, p + \frac{1}{n})$, $\forall n \in \mathbb{Z}^+$.
So $S_{n+1} \subseteq S_n$, $\forall n \in \mathbb{Z}^+$.
Suppose $q \in S_m$ and let $q \neq p$.
Since $q \in S_m = (p - \frac{1}{m}, p + \frac{1}{m})$, then $|p - q| < \frac{2}{m}$.
Then,

$$|p - q| < \frac{2}{m}$$

$$\frac{|p - q|}{2} < \frac{1}{m}$$

$$\frac{2}{|p - q|} > m$$

Pick $l \in \mathbb{Z}^+$ such that $l \geq \frac{2}{|p-q|} > m$.
So $S_l \subset S_m$, $p \in S_l$, and $q \notin S_l$.
$\therefore$ If p is a point, there is a sequence of open intervals $S_1, S_2, S_3, ...$ each containing p such that for each positive integer n, $S_{n+1} \subseteq S_n$, and p is the only point that is in every open interval in the sequence.
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