Math 4590

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Theorem 4
If the sequence $p_1, p_2, p_3, ...$ converges to the point x and y is a point different from x, then y is not a limit point of $\{p_i : i = 1, 2, 3, ...\}$, the range of the sequence.

Proof.
Assume $p_1, p_2, p_3, ...$ converges to x and $y \neq x$.
Since $p_1, p_2, p_3, ...$ converges to x, for every open interval containing x there in an $N \in \mathbb{N}$ such that if $n > N$, then p_n is in the open interval, by definition.
So now we let $\varepsilon = \frac{|x-y|}{2}$, half the distance between x and y.
And let $(x - \varepsilon, x + \varepsilon)$ be an open interval containing x.
Set $M = \{p_1, p_2, p_3, ...\}$ and $D = min\{|y - p_i|\} > 0$, where $1 \leq j \leq N$ and $p_i \neq y$.
Now we let $(\frac{y-D}{2}, \frac{y+D}{2})$ be an open interval containing y.
Since $y \in (\frac{y-D}{2}, \frac{y+D}{2})$ doesn't contain a point of M, then y is not a limit point.
$\therefore$ If the sequence $p_1, p_2, p_3, ...$ converges to the point x and y is a point different from x, then y is not a limit point of $\{p_i : i = 1, 2, 3, ...\}$, the range of the sequence. ∎