

**Theorem 2:** If  $M$  is a finite point set, then  $M$  has a right-most point and a left-most point.

*Proof.* Since  $M$  is finite,  $\exists n > 0$  such that  $M$  has  $n$  points but not  $n + 1$  points. Label them  $M = \{m_1, m_2, m_3, \dots, m_n\}$  in a way that  $m_1 < m_2 < m_3 < \dots < m_n$ .

*Claim 1:*  $m_1$  is a left-most point of  $M$ .

Clearly,  $m_1 \in M$ . All points of  $M$  appear in the inequality

$m_1 < m_2 < m_3 < \dots < m_n$ . We observe that no point of  $M$  is less than  $m_1$ .

Thus,  $m_1$  is the left-most point of  $M$ .

*Claim 2:*  $m_n$  is the right-most point of  $M$ .

Clearly,  $m_n \in M$ . All points of  $M$  appear in the inequality

$m_1 < m_2 < m_3 < \dots < m_n$ . We observe that no point of  $M$  is greater than  $m_n$ .

Thus,  $m_n$  is the right-most point of  $M$ .

$\therefore M$  has a right-most point and a left-most point.  $\square$

**Contrapositive of Theorem 2:** If  $M$  does not have a left-most point or a right-most point, then  $M$  is infinite.

*Proof. (by contrapositive)*

Assume  $M$  does not have a left-most point. Let  $m_1 \in M$ . Then  $\exists m_2 \in M$  such that  $m_2 < m_1$ . Again, since  $M$  does not have a left-most point,  $\exists m_3 \in M$  such that  $m_3 < m_2$ . Continue this process and we achieve the set  $\{m_1, m_2, m_3, \dots\} \subset M$  which is infinite.

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$\therefore$  if  $M$  does not have a right-most point or a left-most point, then  $M$  is infinite.  $\square$