Problem 26: If f is a function which is continuous on $[a, b]$ and $x \in (a, b)$ such that $f(x) > 0$, then there exists an open interval T, containing x, such that $f(t) > 0$ for all $t \in T$.

Proof: Suppose f is such a function. Let $\epsilon > 0$ be small enough so that $0 < \epsilon < f(x)$ (*note: it is sufficient to do this — if $\epsilon > f(x)$, then I would define ϵ_2 such that $0 < \epsilon_2 < \epsilon$ with the property that $\epsilon_2 < f(x)$ and then carry through with the same argument that follows*). Since f is continuous at x, there exists a $\delta > 0$ such that if $|t - x| < \delta$, then $|f(x) - f(t)| < \epsilon$. Since we chose $0 < \epsilon < f(x)$, we observe that

$$|f(x) - f(t)| < \epsilon < f(x).$$

But this just means that "the distance from $f(x)$ to $f(t)$ is less than $f(x)$", or in other words, $f(x) - f(t)$ is not negative (because to be negative would mean that $f(t)$ is farther away from $f(x)$ than $f(x)$), completing the proof. ■