Claim: If $0, a, b$, then $0 < a^2 < b^2$.

Proof. Consider $0 < a < b$. Multiplying by a yields $0 < a^2 < ab$. Multiplying by b yields $0 < ab < b^2$. Combining these inequalities we get $0 < a^2 < ab < b^2$ and thus $0 < a^2 < b^2$. □

Problem 25: Let f be the function such that $f(x) = x^2$ for all numbers x. Show that f is continuous at the point $(2, 4)$.

Proof. Let f be the function such that $f(x) = x^2$ for all numbers x. Let $S = (a, b)$ be an open interval containing $f(2)$. By construction, $b > f(2) = 4$. If $a < 0$, let $S_0 = (0, b) \subset S$, otherwise let $S_0 = S$. Let $S_0 = (c, b)$. We claim that $T = (\sqrt{c}, \sqrt{b})$ will work. Let $t \in T$. Then $\sqrt{c} < t < \sqrt{b}$. Squaring the inequality yields $c < t^2 < b$. Since $t^2 = f(t)$ and $t^2 \in (c, b)$, $f(t) \in S_0 \subset S$. □