

Problem 23: Let f be the function such that $f(x) = 2$ for all numbers $x > 5$, and $f(x) = 1$ for all numbers $x \leq 5$.

1. Show that f is not continuous at the point $(5, 1)$.
2. Show that if t is a number and $t > 5$, then f is continuous at $(t, 2)$.

Proof. 1. Let $S = (0, 2)$. Let T be an open set containing 5. Pick $t \in T$ such that $t > 5$. Then, $f(t) = 2 \notin S$. Thus f is not continuous at $(5, 1)$.

2. Let S be an open set containing 2. Let $T = (5, t + 1)$, where $(t > 5)$. If $w \in T$, then $f(w) = 2 \in S$. Thus f is continuous at $(t, 2)$. \square