

**Problem 20:** Show that if  $M$  is a point set and  $p$  is a point and every closed interval containing  $p$  contains a point of  $M$  different from  $p$ , then  $p$  is a limit point of  $M$ .

**Proof:** Let  $M$  be a set and  $p$  be a point. Suppose all closed intervals containing  $p$  contain a point of  $M$  different from  $p$ , so  $\forall [a, b]$ , if  $p \in [a, b]$ , then  $\exists m \in M \cap [a, b]$  such that  $m \neq p$ . Let  $(c, d)$  be an open interval containing  $p$ . Let  $L$  be any point between  $c$  and  $p$ , and let  $R$  be any point between  $p$  and  $d$ . Notice  $[L, R] \subset (c, d)$ . Since  $[L, R]$  is closed and contains  $p$ ,  $\exists m \in M$  such that  $m \in [L, R] \cap M \subset M \cap (c, d)$ .

Therefore,  $p$  is a limit point of  $M$ .