**Problem 16:** Show that if $c$ is a number and the sequence $p_1, p_2, p_3, ...$ converges to the point $x$, then the sequence $c \cdot p_1, c \cdot p_2, c \cdot p_3, ...$ converges to the point $c \cdot x$.

*Proof.* Suppose $c \in \mathbb{R}$ and $p_1, p_2, p_3, ...$ is a sequence which converges to the point $x$ and let $\epsilon > 0$.
**Case 1:** $c = 0$
Then , $c \cdot x = 0 = c \cdot p_1 = c \cdot p_2 = \cdots = c \cdot p_n$. So, $|(c \cdot x) - (c \cdot p_n)| = 0 < \epsilon, \forall n \in \mathbb{N}^+$. That is, the distance between $c \cdot x$ and $c \cdot p_n$ is always zero, and thus, for any open interval $(a, b)$, where $c \cdot x \in (a, b), p_n \in (a, b) \forall n \geq N$.

**Case 2:** $c \in \mathbb{R} \setminus 0$
Consider $(x - \frac{\epsilon}{|c|}, x + \frac{\epsilon}{|c|})$ for any $\epsilon \in \mathbb{R}$. Since $x \in (x - \frac{\epsilon}{|c|}, x + \frac{\epsilon}{|c|})$, then $\exists N \in \mathbb{Z}^+$ such that $p_n \in (x - \frac{\epsilon}{|c|}, x + \frac{\epsilon}{|c|}), \forall n \geq N$.
Now, $|c \cdot x - c \cdot p_n = |c||x - p_n| < |c|\dfrac{\epsilon}{|c|} = \epsilon$.
Thus, the sequence $c \cdot p_1, c \cdot p_2, c \cdot p_3, ...$ converges to the point $c \cdot x$. □