Real Analysis Problem 14

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February 2020

Problem 14. Show that if the sequence $p_1, p_2, p_3, \ldots$ converges to the point x, and, for each positive integer n, $p_n \neq p_n + 1$, then x is a limit point of the set which is the range of the sequence.

Proof: Let the sequence $p_1, p_2, p_3, \ldots$ converge to the point x and for each positive integer n, $p_n \neq p_{n+1}$. Let M be the set which is the range of the sequence $p_1, p_2, p_3, \ldots$ Let (a, b) be an open interval containing x. Since $p_1, p_2, p_3, ...$ converges to x, $\exists$ a positive integer N s.t. if n is a positive integer and $n \geq N$, then $p_n \in (a, b)$. Also $p_n \in M$ and $p_n \neq p_n + 1$ so $p_n \neq x$, thus x is a limit point of the sequence $p_1, p_2, p_3, \ldots$