

Problem 13: For each positive integer n , let $p_{2n} = 1/(2n-1)$ and let $p_{2n-1} = 1/2n$. Does the sequence p_1, p_2, p_3, \dots converges to 0. Assume ϵ is epsilon.

Proof: Let ϵ be an element of the open interval (a,b) . We define $\epsilon = \min\{|a|, |b|\}$

$$\text{Pick } N > \frac{1}{2\epsilon} + \frac{1}{2} > \frac{1}{2\epsilon}.$$

Let $n > N$.

$$n > \frac{1}{2\epsilon} + \frac{1}{2}$$

$$2n > \frac{1}{\epsilon} + 1$$

$$2n-1 > \frac{1}{\epsilon}$$

$$|0 - p_{2n}| = |0 - \frac{1}{2n}| = \frac{1}{2n-1} < \epsilon.$$

$$n > \frac{1}{2\epsilon}.$$

$$2n > \frac{1}{\epsilon}$$

$$|0 - p_{2n}| = |0 - \frac{1}{2n}| = \frac{1}{2n} < \epsilon$$

Since $\frac{1}{2n-1}$ and $\frac{1}{2n}$ are less than epsilon, p_n is an element of $(-\epsilon, \epsilon)$. Therefore, p_n converges to 0.