

**Problem 11**

For each positive integer  $n$ , let  $p_n = 1 - \frac{1}{n}$ . Show that the sequence  $p_1, p_2, \dots, p_n$  converges to 1.

**Proof:** Let  $(a, b)$  be an open interval containing 1. Choose  $\epsilon = \min\{|1 - a|, |1 - b|\}$ . Pick

$N \in \mathbb{Z}$  such that  $N > \frac{1}{\epsilon}$ . Let  $n > N$ , so  $n > \frac{1}{\epsilon}$  and  $\frac{1}{n} < \epsilon$  by Axiom 6. So,  $\left|1 - \left(1 - \frac{1}{n}\right)\right| = \frac{1}{n} < \epsilon$ .

Hence,  $p_n = 1 - \frac{1}{n}$  converges to 1. ■