

**Problem 8:** Show that if  $H$  is a point set and  $K$  is a point set and every point of  $H$  is a limit point of  $K$  and  $p$  is a limit point of  $H$ , then  $p$  is a limit point of  $K$ .

**Proof:** Let  $(a,b)$  be an open interval containing  $p$ . Since  $p$  is a limit point of  $H$ , every open interval containing  $p$  must contain a point  $a$  different from  $p$ . So let there exist a point  $x$  in  $(a,b)$ . Since  $x$  is in  $H$ , it is a limit point of  $K$ . Also, since  $x$  is a limit point of  $K$ ,  $(a,b)$  must contain a point of  $K$ , called  $c$ . Hence,  $p$  is a limit point of  $K$ .