Real Analysis Problem 6

Ricard Marsal I Castan

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Problem 6. Assume M is a point set such that if P is a point of M, there is a first point to the left of P in M and a first point to the right of P in M. Is it true that M cannot have a limit point?

Proof: By contradiction, suppose P is some limit point of M, L is the first point to the left of P and R is the first point to the right of P. We can create with Ax. 3 a point between L and P ($P - \frac{L+P}{2}$) and another point between P and R ($P + \frac{P+R}{2}$) that doesn't belong to M. The open interval between ($P - \frac{L+P}{2}$, $P + \frac{P+R}{2}$) doesn't have any point in M so P cannot be a limit point of M. Same method can be applied to R and L. Thus, M doesn't have any limit point.