Problem 3

Proof: Let M be a point set and p a limit point of M. Let $(p - 1, p + 1)$ be an open interval. By the definition of limit point, there exists a point a such that $a \in M \cap (p - 1, p + 1)$, and thus M contains one point. Now, pick a p2 that lies between the points p and a, and let d equal the distance between p and p2. Consider the open interval $(p - d, p + d)$. By definition of a limit point, there exists a point b such that $b \in M \cap (p - d, p + d)$. Thus, M contains two points. This process can be repeated to show that if M is a point set containing a limit point, M contains an infinite number of points. ■