Real Analysis Problem 2

Malachi Cowling

January 2020

Problem 2: Show that if M is the closed interval [a,b], and p is not in M, then p is not a limit point of M.

Limit Point: X is a limit point of M means every open interval containing p contains a point of M different from p.

Solution:

Goal: We need to show that there is an open interval (c,d) such that $p \in (c, d)$ and no point of M is in (c,d).

Proof: Since $p \notin M$, as stated in the hypothesis, it is not the case that $a \leq p \leq b$. By Axiom 2, we know that p must be less than a or greater than b.

Case 1: $p < a$
Pick c = p - 1. Axiom 3 tells us that there exists a d between p and a. Let $(c, d) = x : c < x < d$. We know $c < p$ because $c = p - 1 < p$. We know $p < d$ because d is between p and a.
Therefore, $p \in (c, d)$. Since the right endpoint of (c,d) is to the left of a, (c,d) does not contain any points in M.

Case 2: $p > b$
Pick d = p + 1. By Axiom 3, we know that there exists a c between p and b. We know that $c < p$ because c is between p and b. We also know that $p < d$ because $d = p + 1 > p$.
Therefore, $p \in (c, d)$. Since the left endpoint of (c,d) is to the right of b, (c,d) does not contain any points in M.

By Cases 1 and 2, if M is the closed interval [a,b], and p is not in M, then p is not a limit point of M.