Written HW13 – MATH 3503 Fall 2020

Due by 25 November for timely completion credit

Throughout, suppose that $\vec{F}$ is a three-dimensional vector field. Recall Stokes' theorem which says if C is the boundary curve of an orientable surface S, then

$$\oint_C \vec{F} \cdot \mathrm{d}\vec{r} = \iint_S \mathrm{curl}(\vec{F}) \cdot \mathrm{d}\vec{S}.$$

Also recall the divergence theorem which says that if E is a region in space with boundary surface S, then

$$\iint_S \vec{F} \cdot \mathrm{d}\vec{S} = \iiint_V \mathrm{div}\vec{F}\, \mathrm{d}V.$$

(1) Let $\vec{F} = \langle y^2, x, z^2 \rangle$ and let S be the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$. Compute both sides of Stokes' theorem for this setup.

(2) Plot the surface and the curve involved in (1) and attach the picture to your submission.

(3) Let $\vec{F} = \langle x^2, xy, z \rangle$ and let E be the solid bounded by both the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane. Compute both sides of the divergence theorem for this setup.