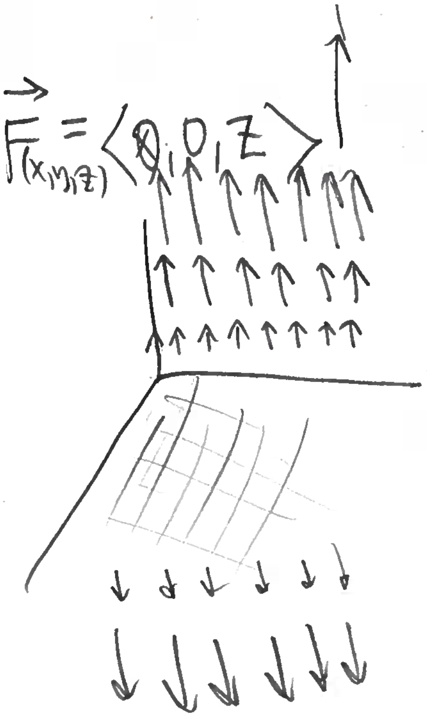
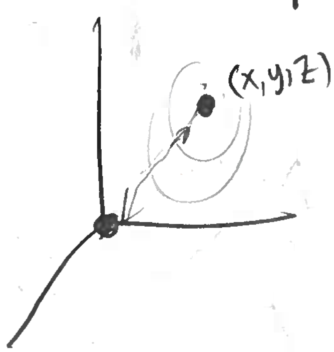
Ex: $\vec{F}_{(x,y,z)} = \langle 0, 0, z \rangle$



Ex: Electric charge Q at $(0,0,0)$

Coulomb's law $\rightarrow$ electric force $\vec{F}$ exerted by charge q at (x,y,z)

$$\vec{F}(x,y,z) = \frac{\epsilon q Q}{\|\langle x,y,z\rangle\|^2} \langle x,y,z \rangle$$



like charge : $qQ > 0$ ~ repulsive

opp charge : $qQ < 0$ ~ attractive

Recall gradient

$$\nabla f(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

$$\nabla f(x,y,z) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

How does f relatee to ∇f?

Ex: $f(x,y) = x^2y - y^3$

Recall: level curves

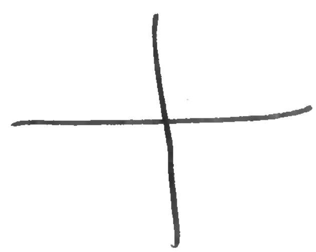
$k = z = f(x,y)$

$k=3$

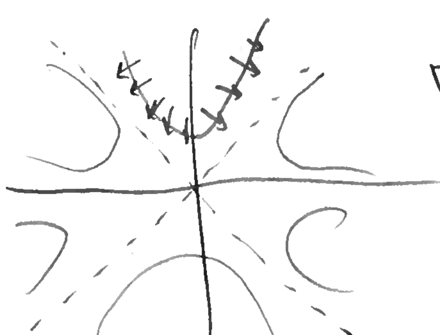
$3 = x^2y - y^3 \rightarrow$

$k=2$

$2 = x^2y - y^3$



fn gen k



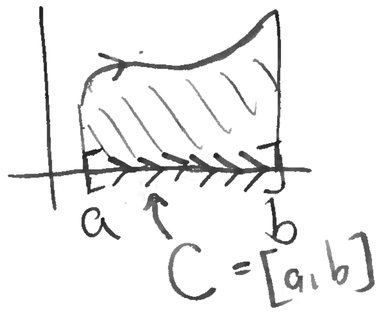
$$\nabla f = \langle 2x, x^2 - 3y^2 \rangle$$

Def: Vector field $\vec{F}$ is called conservative if it is the gradient field of a function f.

i.e. $\vec{F}$ conservative $\longleftrightarrow$ $\boxed{\vec{F} = \nabla f}$

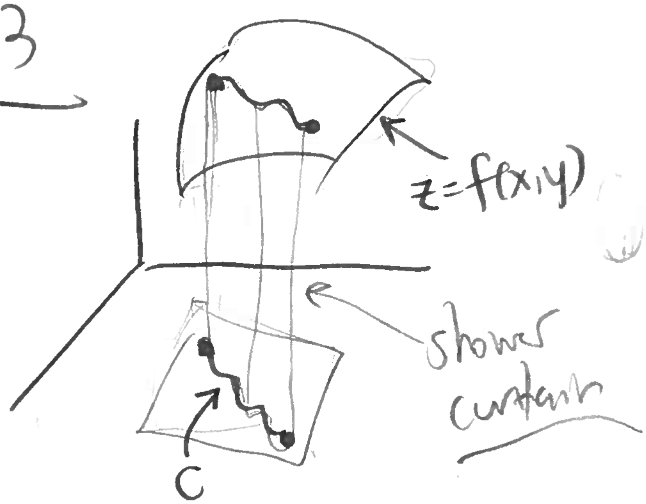
Line Integrals ("path integral")

Calc 1-2 integral



$$\int_a^b f(x)\,dx$$

Calc 3



$$\int_C f(x,y)\,ds = \text{Area of the shower curtain}$$

$$\overset{def}{=} \int_a^b f(x(t),y(t))\sqrt{(x')^2+(y')^2}\,dt$$

$$= \int_a^b f(\vec{r}(t))\,\|\vec{r}\,'(t)\|\,dt$$

parametrize curve C as

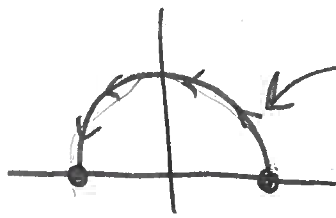
$$\vec{r}(t) = \begin{cases} x = x(t) \\ y = y(t) \\ a \le t \le b \end{cases}$$

$$\|\langle a,b\rangle\| \overset{\text{def}}{=} \sqrt{a^2+b^2}$$

Ex: Evaluate

$\int_C 2+x^2y\,ds$ where C is upper half of the unit circle.

Soln:



$$\begin{cases}\vec{r}(t)=\langle\cos(t),\sin(t)\rangle \\ 0\le t\le\pi\end{cases} \qquad \vec{r}\,'(t)=\langle -\sin(t),\cos(t)\rangle$$

So,

$$\int_C 2+x^2y\,ds = \int_0^\pi \underbrace{\left[2+\cos^2(t)\sin(t)\right]}_{\text{"}f(\vec{r}(t))\text{"}}\underbrace{\sqrt{(-\sin(t))^2+\cos^2(t)}}_{\sqrt{1}}\,dt$$

$t=0\to u=1$

$t=\pi\to u=-1$

$$=\int_0^\pi 2\,dt+\int_0^\pi \cos^2(t)\sin(t)\,dt$$

$u=\cos(t)$

$du=-\sin(t)dt$

$-du=\sin(t)dt$

$$-\int_a^b=\int_b^a$$

$$=2\pi-\int_1^{-1}u^2\,du$$

$$=2\pi+\int_{-1}^{1}u^2\,du$$

$$=2\pi+\frac{u^3}{3}\Big|_{-1}^{1}=2\pi+\left[\frac{1}{3}-\left(-\frac{1}{3}\right)\right]$$

$$=2\pi+\frac{2}{3}$$