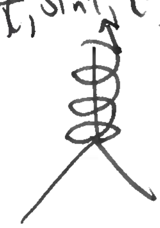
What we've done

Calc 1-2 : $f: \mathbb{R} \to \mathbb{R}$

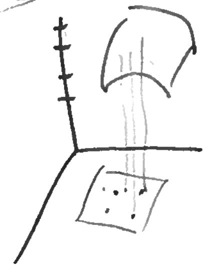
early calc 3 : space curves

$f: \mathbb{R} \to \mathbb{R}^n$ $\vec{F}(t) = \langle t, t^2, 1-t \rangle$

$\langle \cos t, \sin t, t \rangle$



mid calc 3 : $f: \mathbb{R}^n \to \mathbb{R}$



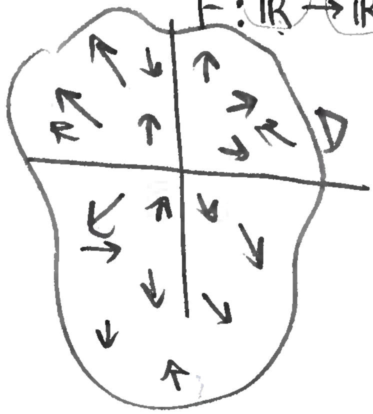
now : $f: \mathbb{R}^m \to \mathbb{R}^n$

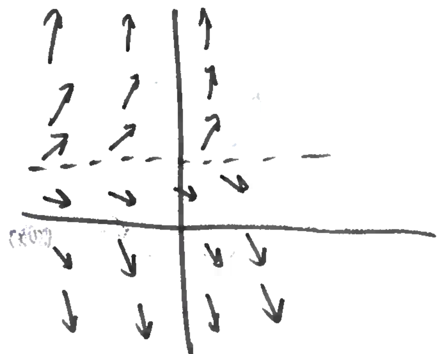
$\nabla f(x,y) = \langle f_x, f_y \rangle$

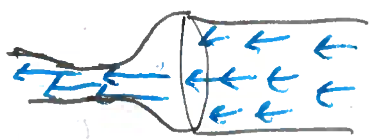
$\nabla f: \mathbb{R}^2 \to \mathbb{R}^2$

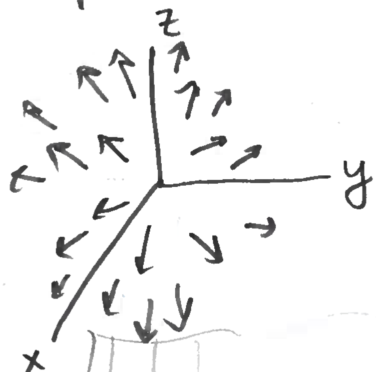
Def: A vector field is a function $\vec{F}$ that assigns to each point in D a vector.

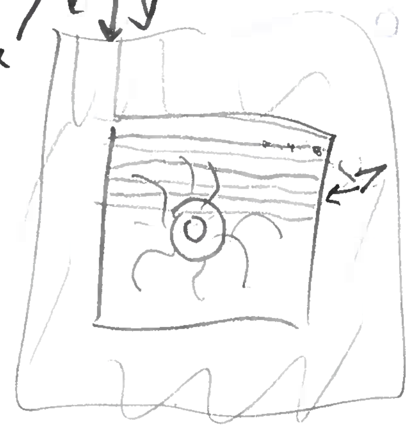
$F: \mathbb{R}^2 \to \mathbb{R}^2$











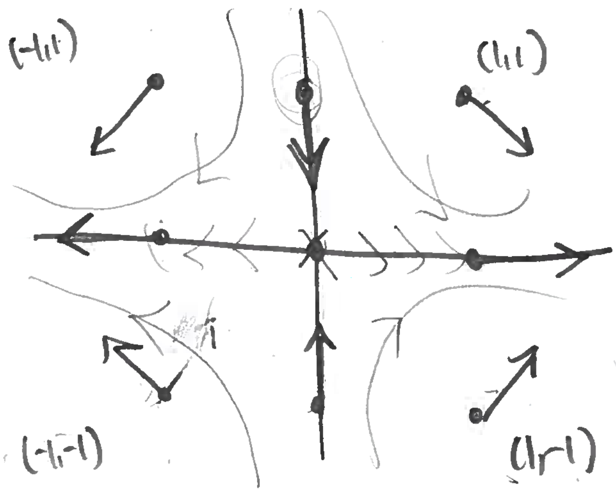
Usually we write vector fields in terms of component functions:

$$\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$$

$$\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$$

Ex: Sketch vector field

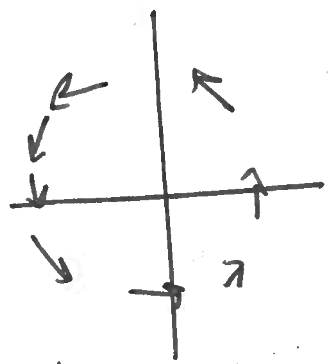
$$\vec{F}(x,y) = \langle x, -y \rangle$$

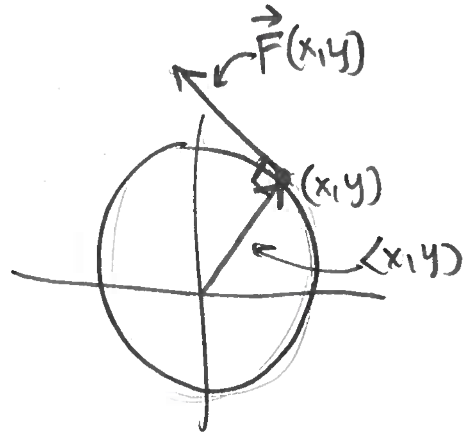


$\langle x, -y \rangle$

(x,y)	$\vec{F}(x,y)$	
$(-1,1)$	$\langle -1,-1 \rangle$	$\swarrow$
$(0,1)$	$\langle 0,-1 \rangle$	$\downarrow$
$(1,1)$	$\langle 1,-1 \rangle$	$\searrow$
$(-1,0)$	$\langle -1,0 \rangle$	$\leftarrow$
$(0,0)$	$\langle 0,0 \rangle$	$\times$
$(1,0)$	$\langle 1,0 \rangle$	$\rightarrow$
$(-1,-1)$	$\langle -1,1 \rangle$	$\nwarrow$
$(0,-1)$	$\langle 0,1 \rangle$	$\uparrow$
$(1,-1)$	$\langle 1,1 \rangle$	$\nearrow$

Ex: $\vec{F} = \langle -y, x \rangle$





seems like each arrow is tangent to a circle

Can we validate this?

$$\langle x,y \rangle \cdot \vec{F}(x,y) = \langle x,y \rangle \cdot \langle -y,x \rangle$$
$$= -xy + xy$$
$$= 0 \sim \text{orthogonal}$$