Cylindrical coords

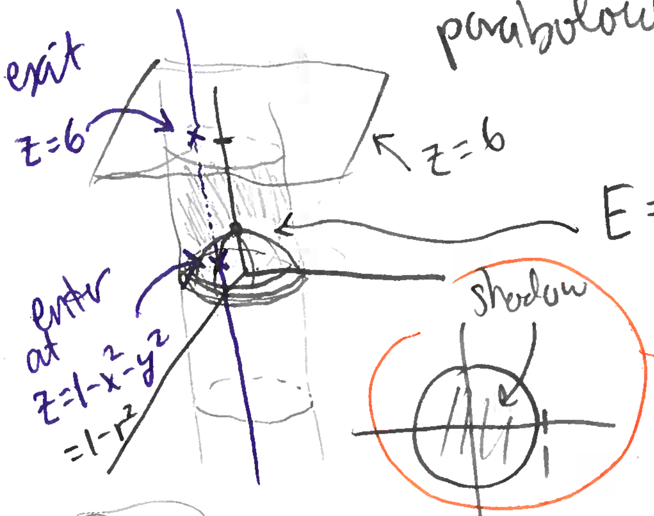
"lazy generaliz. of polar"

$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \\ z = z \end{cases}$$

$$dV \xrightarrow{\text{cyl.}} \overset{\substack{\text{extra}\\ \downarrow}}{\textcircled{r}}\, dr\, d\theta\, dz$$

$r\, dz\, dr\, d\theta$

Ex: Compute $\iiint_E \textcircled{x}\, dV$ where E is region inside cylinder $x^2+y^2=1$, below plane $z=6$, + above paraboloid $z = 1-x^2-y^2$.



$$E = \left\{ (r,\theta,z) : \begin{array}{l} 1-r^2 \le z \le 6 \\ 0 \le r \le 1 \\ 0 \le \theta \le 2\pi \end{array} \right\}$$

$$1-x^2-y^2 = 1-r^2\cos^2(\theta) - r^2\sin^2(\theta)$$
$$= 1-r^2(\cos^2(\theta)+\sin^2(\theta))$$
$$= 1-r^2$$

So,

$$\iiint_E x\, dV$$
$$= \int_0^{2\pi}\int_0^1\int_{1-r^2}^6 (r\cos(\theta))\, \overset{\substack{\text{extra}\\ \downarrow}}{\textcircled{r}}\, dz\, dr\, d\theta$$

$$= \ldots$$
$$\overset{?}{=} 0$$

$$\sin(\theta)\Big|_0^{2\pi}$$

EX: Find region of integration + compute

$$\underbrace{\int_{-2}^{2}}_{\text{numbers}}\ \underbrace{\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}}_{\text{curves}}\ \underbrace{\int_{\sqrt{x^2+y^2}}^{2} x^2+y^2}_{\text{surfaces}}\, dz\, dy\, dx$$

$\int y^2\sqrt{x^2+y^2}\,dy$

↑

NASTY

Shadow

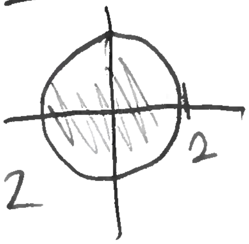
↳ enter on curve

$y=-\sqrt{4-x^2} \searrow$

exit on $\qquad y^2=4-x^2 \qquad \Longrightarrow$

$y=\sqrt{4-x^2} \nearrow \qquad \boxed{x^2+y^2=4} \leftarrow$ circle of rad 2

shadow



Surfaces

↳ enter on $z=\sqrt{x^2+y^2} \leftarrow z^2-x^2-y^2=0$ ← cone

exit on $z=2$ ← plane

enter on cyl coord

$z=\sqrt{x^2+y^2}=\sqrt{r^2}=r$

interior of this cone ← $C=\left\{(r,\theta,z):\ \begin{array}{l} r\le z\le 2 \\ 0\le r\le 2 \\ 0\le\theta\le 2\pi \end{array}\right\}$

$$\Longrightarrow \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} x^2+y^2\,dz\,dy\,dx = \int_0^{2\pi}\int_0^2\int_r^2 r^2\cdot \overset{\text{extra}}{\overset{\downarrow}{r}}\,dz\,dr\,d\theta$$

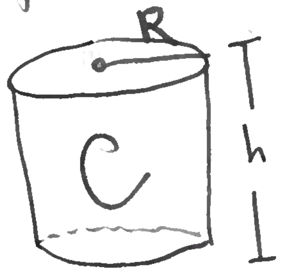
$\frac{2}{3}r^3-\frac{r^4}{4}$

$$= \int_0^{2\pi}\int_0^2 r^2(2-r)\,dr\,d\theta$$

$2r^2-r^3$

$$= \int_0^{2\pi}\left(\frac{16}{3}-\frac{16}{4}\right)-(0)\,d\theta = 2\pi\left(\frac{16}{3}-\frac{16}{4}\right)$$

Volume of a right circ. cylinder

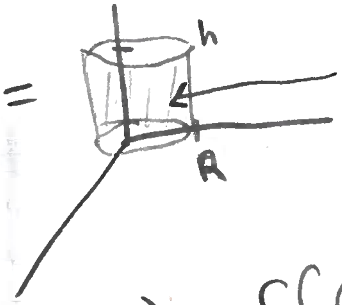


Vol = $\pi R^2 h$

See using $\iiint$:

Vol of region bdd by $x^2+y^2=R^2$, $z=0$, $z=h$.

$C =$



$$\mathcal{C}=\left\{(r,\theta,z):\ \begin{matrix}0\le z\le h\\ 0\le r\le R\\ 0\le\theta\le 2\pi\end{matrix}\right\}$$

$$\mathrm{Vol}(C)=\iiint_C 1\,dV$$

$$=\int_0^{2\pi}\int_0^R\int_0^h \overset{\text{extra } r}{\boxed{r}}\,dz\,dr\,d\theta$$

$$=\boxed{h}\int_0^{2\pi}\int_0^R \overset{r^2/2}{\boxed{r}}\,dr\,d\theta$$

$$=h\int_0^{2\pi}\frac{R^2}{2}\,d\theta=\frac{hR^2}{2}\overset{=2\pi}{\boxed{\int_0^{2\pi} 1\,d\theta}}$$

$$=\pi R^2 h$$