

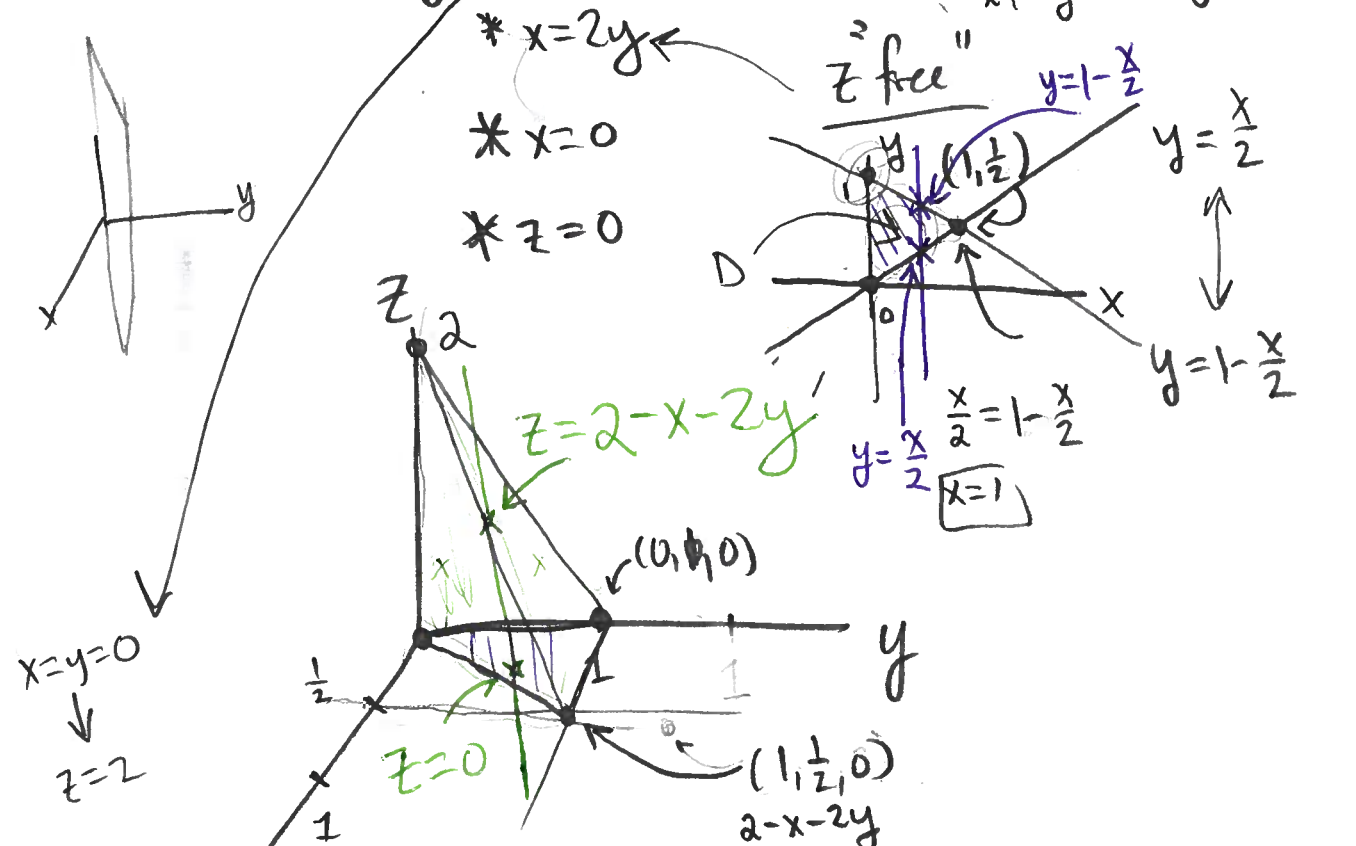
~~Ex:~~ earlier:  $\text{Area}(D) = \iint_D 1 dA$

Now:  $\text{Vol}(D) = \iiint_D 1 dA$

Ex: Find volume of tetrahedron T

bdd by  $x+2y+z=2$

at  $z=0$   
 $x+2y=2 \rightarrow y = \frac{2-x}{2}$



$\text{Vol}(T) = \iiint_T 1 dA = \iiint_D 1 dz$

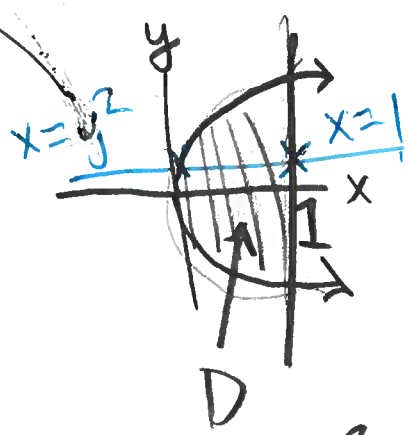
$= \int_0^1 \int_{\frac{x}{2}}^{1-\frac{x}{2}} \int_0^{2-x-2y} 1 dz dy dx$

$= \dots = \frac{1}{3}$

(2)

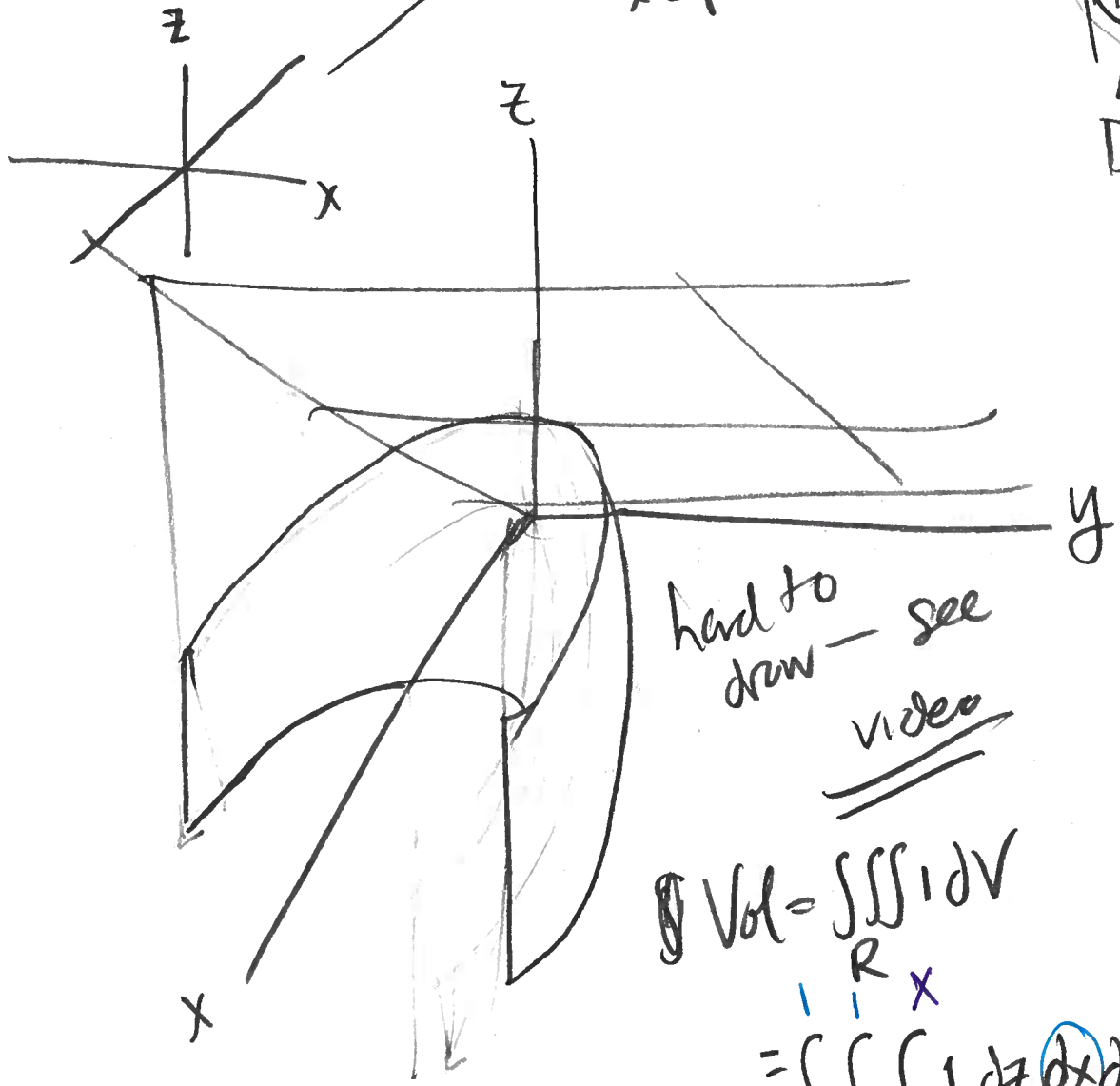
Ex: Find volume of region bdd by  
parabolic cylinder  $x=y^2$

+ planes  
 $x=z$   
 $z=0$   
 $x=1$



$$y^2=1$$

$$y=\pm 1$$



hard to draw - see  
video

$$\text{Vol} = \iiint_R 1 \, dV$$

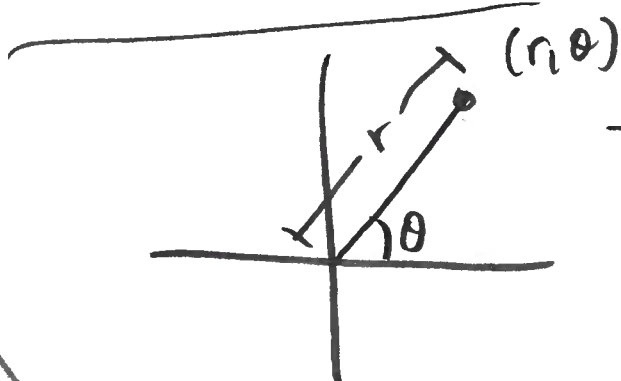
$$= \int_{-1}^1 \int_{y^2}^1 \int_0^x 1 \, dz \, dx \, dy$$

# Cylindrical coordinates

3

"lazy extension of polar to 3D"

Recall polar (2D)



$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

cyl coords

$(r, \theta, z)$

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases}$$

Ex: cyl point  $(2, \frac{\pi}{2}, 1)$

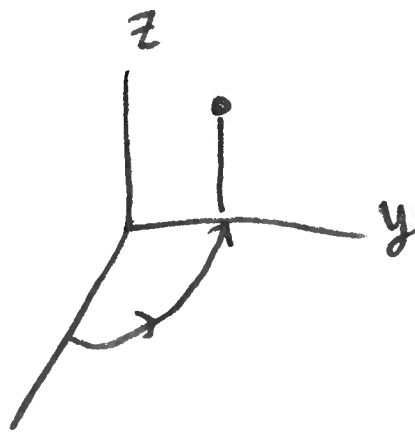


rect coords?

$$x = 2 \cos\left(\frac{\pi}{2}\right) = 0$$

$$y = 2 \sin\left(\frac{\pi}{2}\right) = 2$$

$$z = 1$$



Ex: if  $(3, -3, 4)$  rect coords  
↓ find cyl coords

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases}$$

$$x^2 + y^2 = r^2$$

$$r = +\sqrt{x^2 + y^2}$$

$$\frac{y}{x} = \frac{r \sin \theta}{r \cos \theta} = \tan(\theta)$$

$$z = 4$$

$$r = \sqrt{3^2 + (-3)^2} = \sqrt{18}$$

$$(x, y) = (3, -3) \rightarrow \text{in QIV}$$

∥∥

$$\theta = \arctan\left(\frac{-3}{3}\right)$$

$$= \arctan(-1)$$

$$= -\frac{\pi}{4}$$

$$\theta = \begin{cases} \arctan\left(\frac{y}{x}\right); & (x, y) \text{ is in QI or QIV} \\ \pi + \arctan\left(\frac{y}{x}\right) & (x, y) \text{ is in QII or QIII} \end{cases}$$

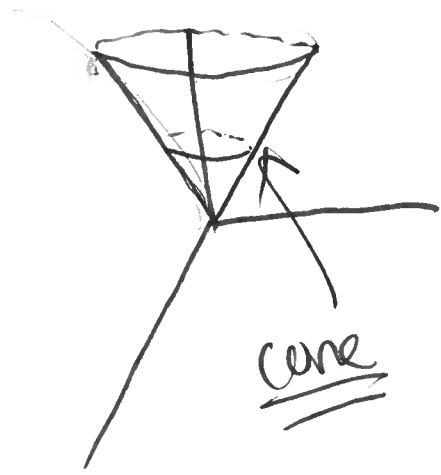


Ex: Describe the surface whose cyl coords

are  $z = r$   
↑ height = radius

$\theta$  free to spin

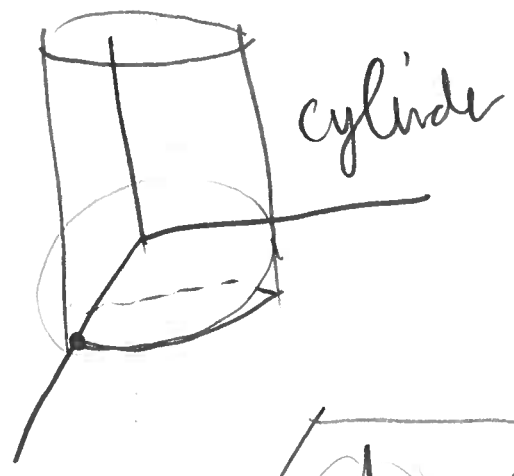
Soln:



Ex:  $\theta = C \leftarrow$  constant



Ex:  $r = C$



Ex:  $z = C$

