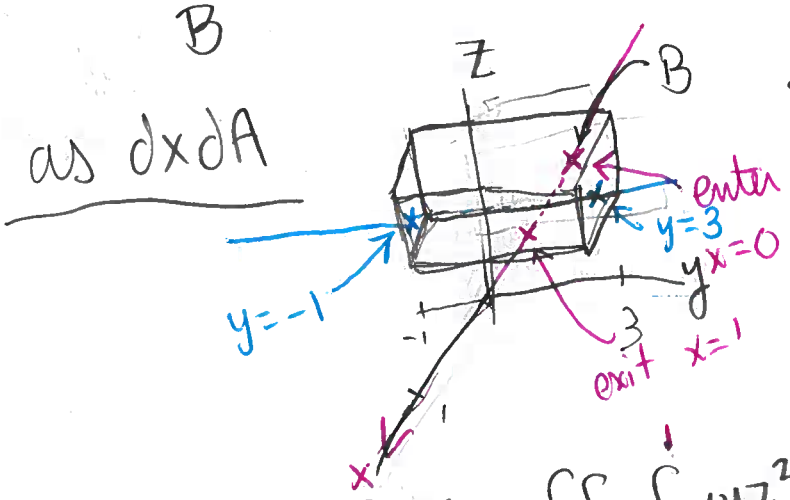
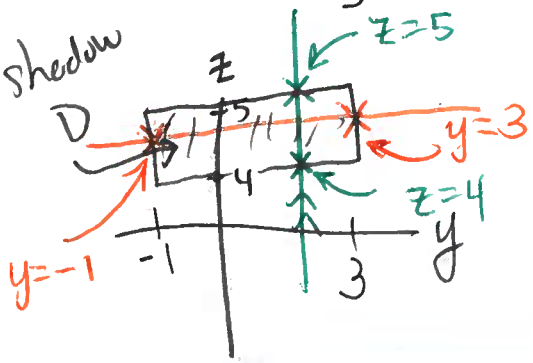
(continued)

Ex: $\iiint_B xyz^2\,dV$, where $B = \{(x,y,z): \begin{matrix} 0\le x\le 1 \\ -1\le y\le 3 \\ 4\le z\le 5 \end{matrix}\}$

as dx dA





$$\iiint_B xyz^2\,dV = \iint_D \int_0^1 xyz^2\,dx\,dA$$

dx dz dy

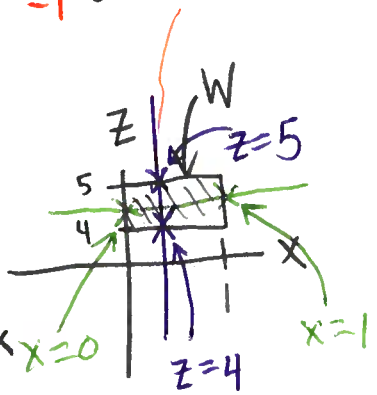
$$\int_{-1}^{3}\int_{4}^{5}\int_{0}^{1} xyz^2\,dx\,dz\,dy$$

dx dy dz

$$\int_{4}^{5}\int_{-1}^{3}\int_{0}^{1} xyz^2\,dx\,dy\,dz$$

as dy dA

$$\iiint_B xyz^2\,dV = \iint_W \int_{-1}^{3} xyz^2\,dy\,dA$$



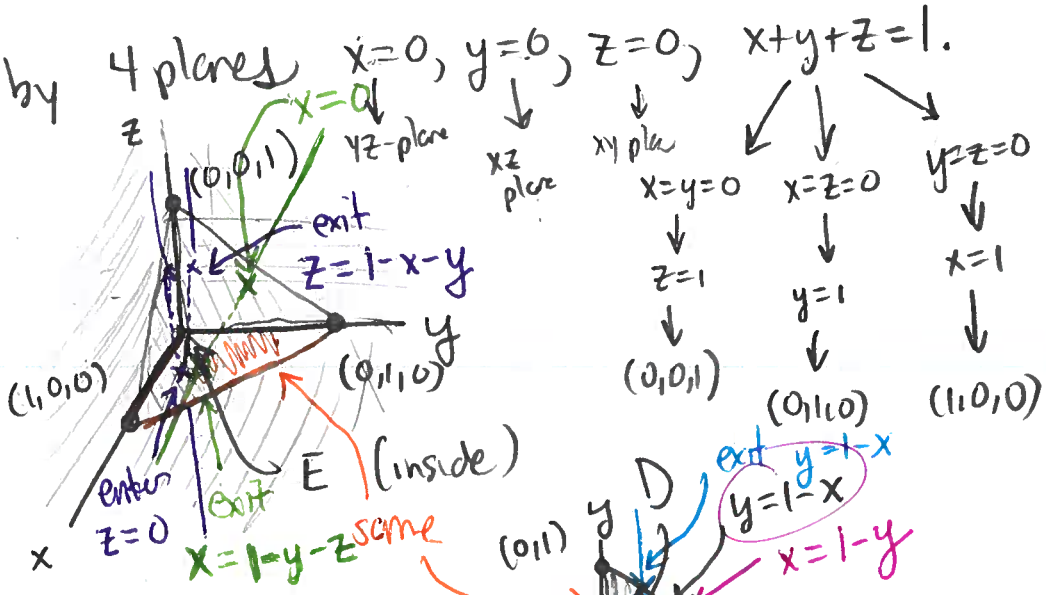
dy dx dz

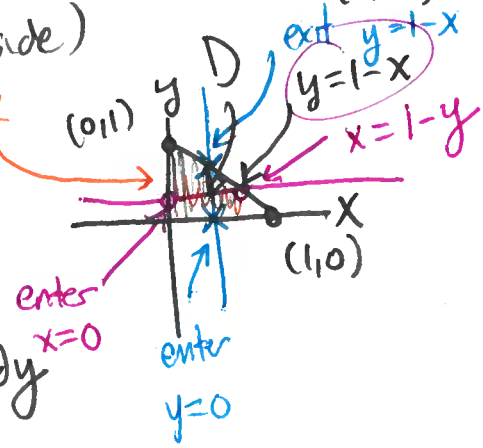
$$\int_{4}^{5}\int_{0}^{1}\int_{-1}^{3} xyz^2\,dy\,dx\,dz$$

$$\int_{0}^{1}\int_{4}^{5}\int_{-1}^{3} xyz^2\,dy\,dz\,dx$$

Ex: $\iiint_E zy\,dV$, where E is tetrahedron

bdd by 4 planes $x=0$, $y=0$, $z=0$, $x+y+z=1$.





$$\iiint_E zy\,dV = \iint_D \int_0^{1-x-y} zy\,\underline{dz}\,dA$$

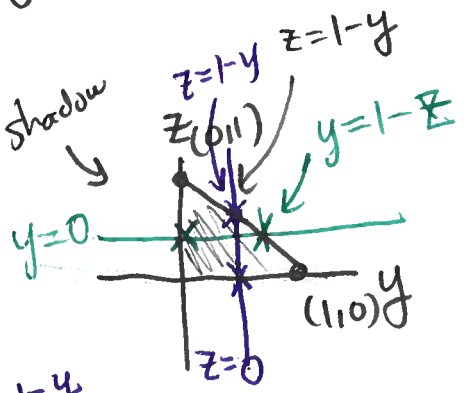
$dz\,dA$

$dz\,dy\,dx$:
$$= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} zy\,dz\,\underline{dy}\,dx$$

$dz\,dx\,dy$:
$$= \int_0^1 \int_0^{1-y} \int_0^{1-x-y} zy\,dz\,\underline{dx}\,dy$$

$dx\,dA$

$$\iiint_E zy\,dV = \iint_W \int_0^{1-y-z} zy\,\underline{dx}\,dA$$



$dx\,dy\,dz$:
$$= \int_0^1 \int_0^{1-z} \int_0^{1-y-z} yz\,dx\,\underline{dy}\,dz$$

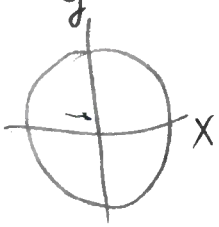
$dx\,dz\,dy$:
$$= \int_0^1 \int_0^{1-y} \int_0^{1-y-z} yz\,dx\,\underline{dz}\,dy$$

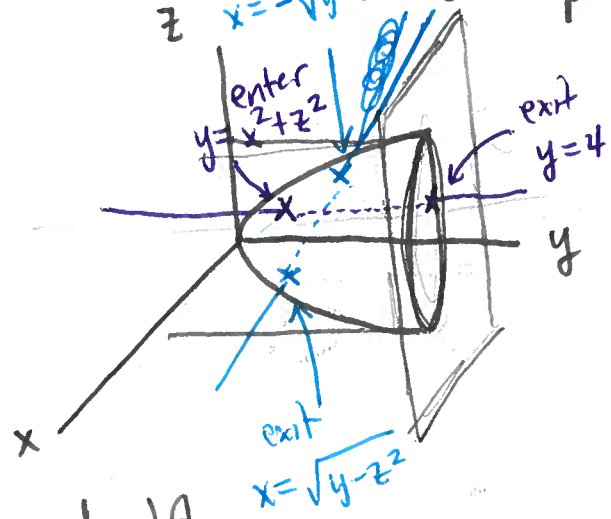
Ex: $\iiint_E \sqrt{x^2+z^2}\,dV$ E bdd by paraboloid $y = x^2+z^2$ ($x = \pm\sqrt{y-z^2}$) and plane $y=4$

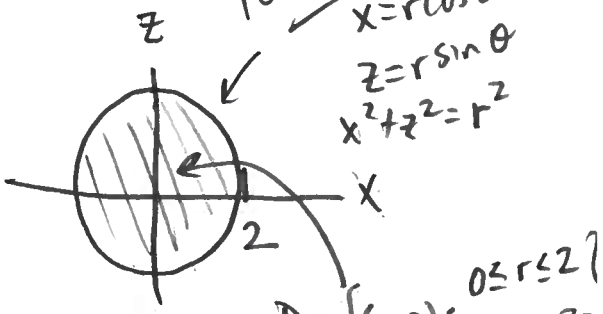
$x^2+z^2=4$

$x = r\cos\theta$

$y = r\sin\theta$







$D = \{(r,\theta): 0\le r\le 2,\ 0\le\theta\le 2\pi\}$

as dy dA

$$\iiint_E \sqrt{x^2+z^2}\,dV = \iint_D \int_{x^2+z^2}^{4} \sqrt{x^2+y^2}\,dy\,dA$$

$$= \int_0^{2\pi}\int_0^2 \int_{x^2+z^2}^{4} r^2\sqrt{r^2\cos^2(\theta)+y^2}\,(r)\,dy\,dr\,d\theta$$

nasty!

as dx dA

$$\iiint_E \sqrt{x^2+z^2}\,dV = \iint_W \int_{-\sqrt{y-z^2}}^{\sqrt{y-z^2}} \sqrt{x^2+z^2}\,dx\,dA$$

$z = \pm\sqrt{y}$

$y = z^2$

$y = 4$

$z = \sqrt{y}$

$z = -\sqrt{y}$

$z^2 = 4$

$z = \pm 2$

$dx\,dy\,dz$

$$\int_{-2}^{2}\int_{z^2}^{4}\int_{-\sqrt{y-z^2}}^{\sqrt{y-z^2}} \sqrt{x^2+z^2}\,dx\,dy\,dz$$

$dx\,dz\,dy$

$$\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_{-\sqrt{x^2+z^2}}^{\sqrt{x^2+z^2}} \sqrt{x^2+z^2}\,dx\,dz\,dy$$