Probability distributions

A "random variable" X has an associated "density".

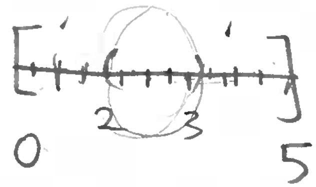
$$\int_1^{\infty}$$

Ex: r.v. X that takes values in $[0,5]$ & it is equally likely that you get any number there equally

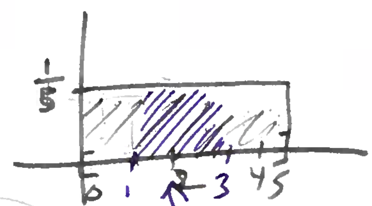
probability is a number in $[0,1]$

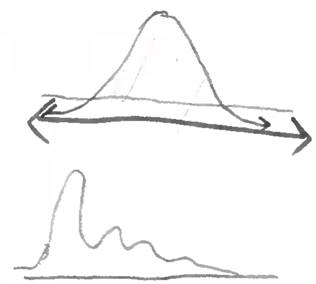
$p=0.5 \leftrightarrow 50\%$

$p=0.21 \leftrightarrow 21\%$



distribution





What is prob that the number I get is in $[1,3]$?

This area

this probability] same as [

$$P(X \in [1,3]) = \frac{2}{5}$$

$$= \int_1^3 \frac{1}{5}\,dx =$$

Most famous: normal distribution

$$\int_0^t e^{-x^2}\,dx = \operatorname{erf}(t)$$

$\sigma = 1,\ \mu = 0 \implies$ "standard normal"

$$-\int_a^b = \int_b^a$$

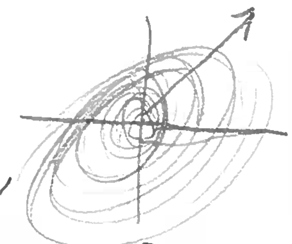
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\int_1^2 x\,dx = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

Must be that $\boxed{\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx = \sqrt{2\pi}}$

$$\int_1^2 y\,dy = \frac{y^2}{2}\Big|_1^2 = \frac{4}{2} - \frac{1}{2}$$

$$\int_a^a f(x)\,dx = 0$$



$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx = \sqrt{\left(\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}\,dy\right)}$$

$$= \sqrt{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{(x^2+y^2)}{2}}\,dx\,dy}$$

magical! $\mathbb{R}^2 = \left\{(r,\theta) : \begin{array}{l} 0 < r < \infty \\ 0 < \theta \le 2\pi \end{array}\right\}$

$x^2 + y^2 = r^2$

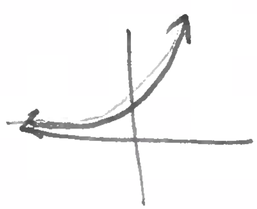
$$= \sqrt{\iint_{\mathbb{R}^2} e^{-\frac{(x^2+y^2)}{2}}\,dA} = \sqrt{\int_0^{2\pi}\int_0^{\infty} e^{-\frac{r^2}{2}}\, r\,dr\,d\theta}$$

$u = -\frac{r^2}{2}$ $\quad r = 0 \to u = 0$

$\quad r = \infty \to u = -\infty$

$du = -r\,dr$

$-du = r\,dr$



$$= \sqrt{-\int_0^{2\pi}\int_0^{-\infty} e^u\,du\,d\theta}$$

$$= \sqrt{\int_0^{2\pi} e^u\Big|_{u=-\infty}^{u=0}\,d\theta} = \sqrt{\int_0^{2\pi} \underset{1}{e^0} - \underbrace{\lim_{t\to-\infty} e^t}_{=0}\,d\theta} = \sqrt{\int_0^{2\pi} 1\,d\theta} = \sqrt{2\pi}$$