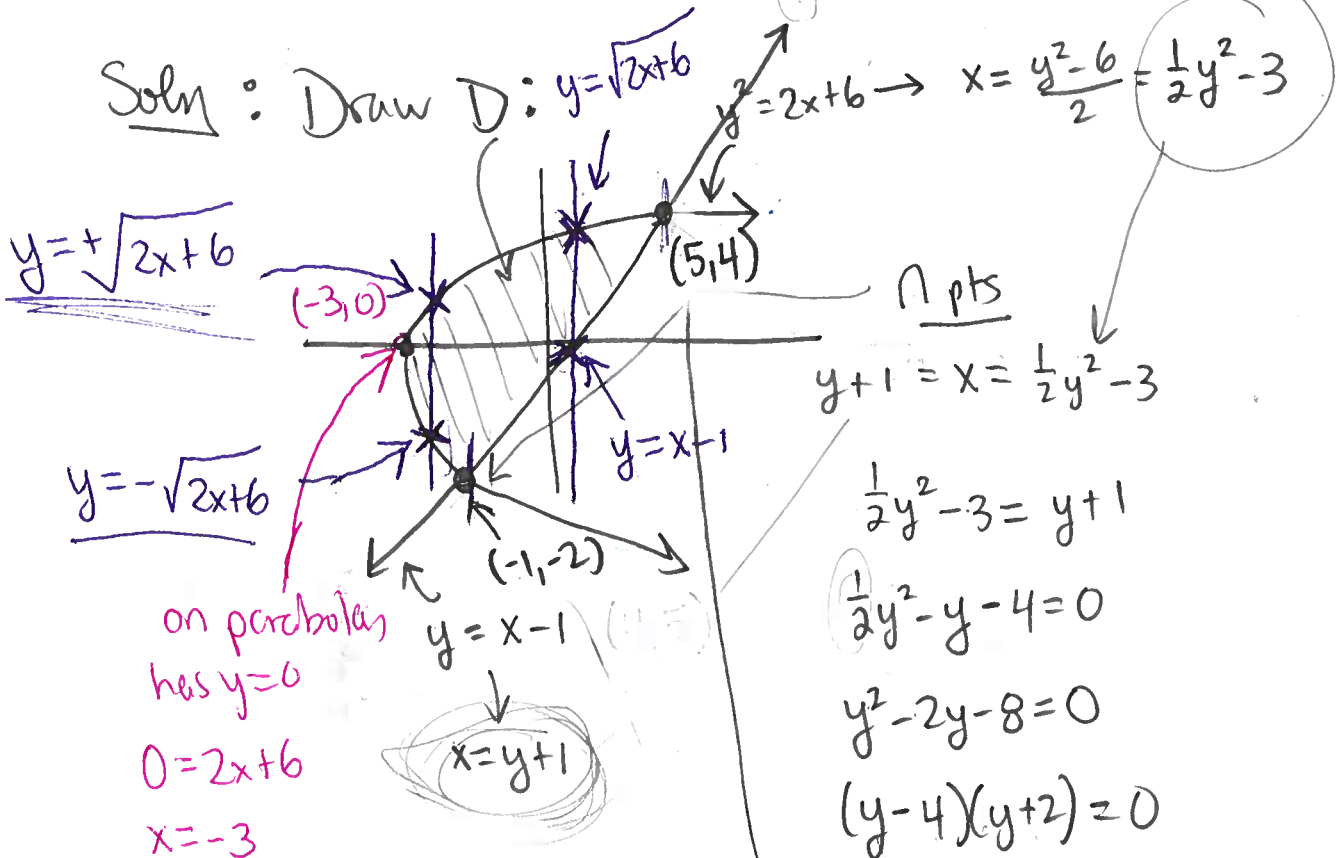
Ex: Calculate $\iint_D xy\,dA$ where D is bdd by $y = x-1$ and parabola $y^2 = 2x+6$. $\leftarrow y = \pm\sqrt{2x+6}$

Soln: Draw D: $y = \sqrt{2x+6}$



$y^2 = 2x+6 \rightarrow x = \frac{y^2-6}{2} = \frac{1}{2}y^2 - 3$

$\cap$ pts

$y+1 = x = \frac{1}{2}y^2 - 3$

$\frac{1}{2}y^2 - 3 = y+1$

$\frac{1}{2}y^2 - y - 4 = 0$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4, -2$

$y=4 \rightarrow x=5 \Rightarrow (5,4)$

$y=-2 \rightarrow x=-1 \Rightarrow (-1,-2)$

on parabola has $y=0$

$0 = 2x+6$

$x = -3$

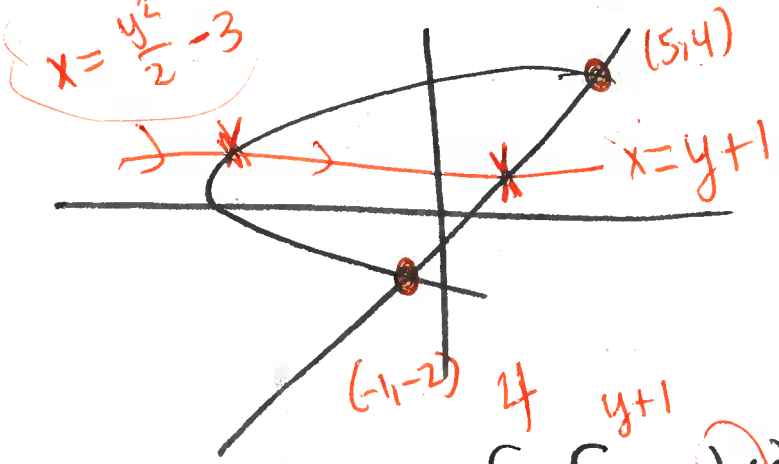
$y = x-1$

$x = y+1$

as $dy\,dx$

$$\iint_D xy\,dA = \int_{-3}^{-1}\int_{-\sqrt{2x+6}}^{\sqrt{2x+6}} xy\,dy\,dx + \int_{-1}^{5}\int_{x-1}^{\sqrt{2x+6}} xy\,dy\,dx$$

as $dx\,dy$ $\quad y^2 = 2x+6$



$$\iint_D xy\,dA = \int_{-2}^{4}\int_{\frac{y^2}{2}-3}^{y+1} xy\,dx\,dy$$

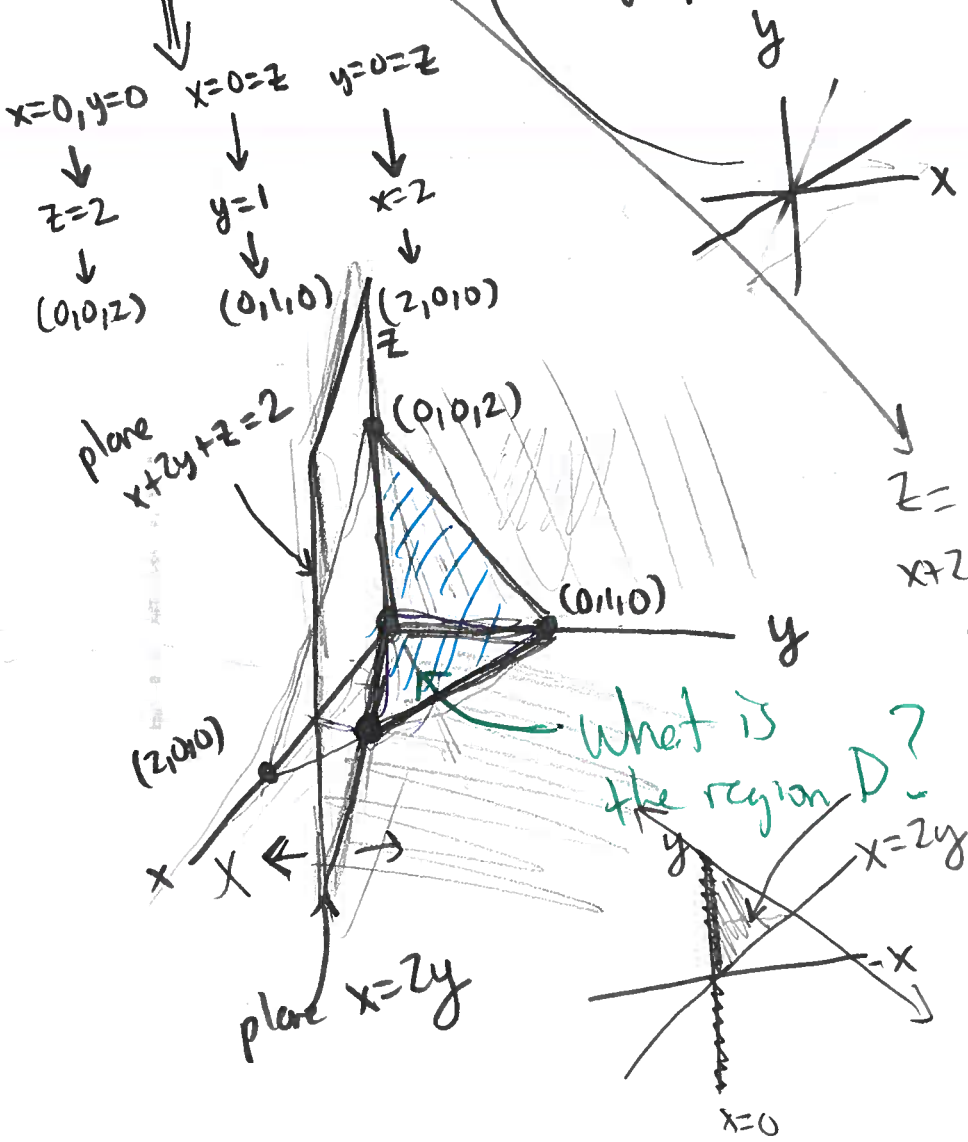
Ex: Find volume of tetrahedron bdd by $x+2y+z=2$, $x=2y$, $x=0$, and $z=0$.

$x+2y+z=2$: plane; $x=2y$: plane; $x=0$: yz-plane; $z=0$: xy-plane

$x=0, y=0 \Rightarrow z=2 \Rightarrow (0,0,2)$

$x=0=z \Rightarrow y=1 \Rightarrow (0,1,0)$

$y=0=z \Rightarrow x=2 \Rightarrow (2,0,0)$

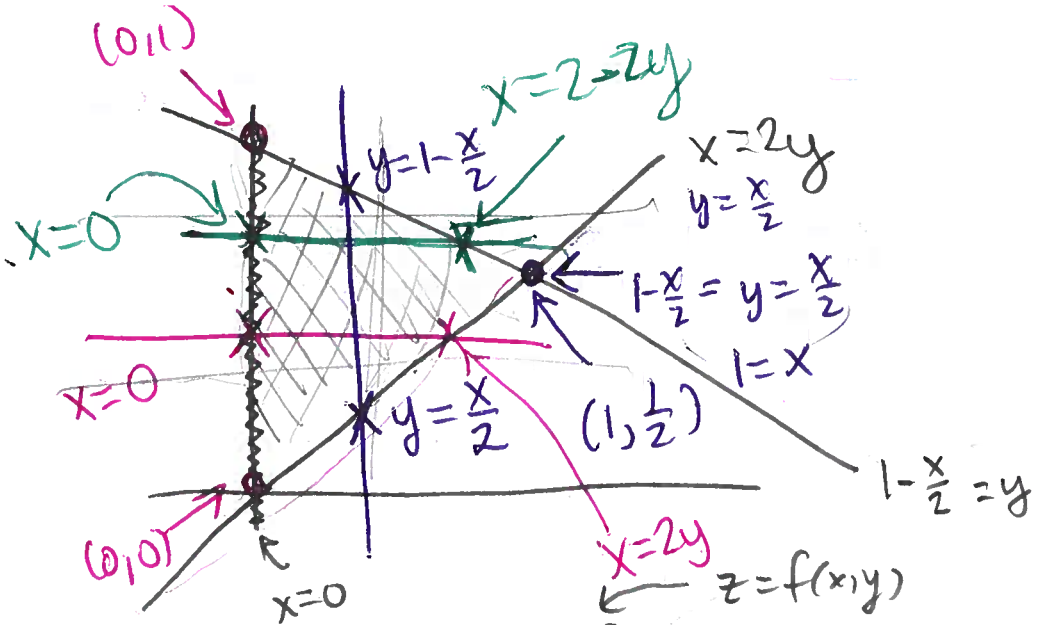


$z=0$

$x+2y=2$

$y=\frac{2-x}{2}=1-\frac{x}{2}$

What is the region D?



Vol = Volume under plane $x+2y+z=2$ $\quad = 2-x-2y$
+ above D

$$= \iint_D 2-x-2y \, dA$$

dydx easier

$$= \int_0^1 \int_{\frac{x}{2}}^{1-\frac{x}{2}} 2-x-2y \, dy\,dx$$

as dxdy

$$= \int_{1/2}^{1} \int_{0}^{2-2y} 2-x-2y \, dx\,dy + \int_{0}^{1/2} \int_{0}^{2y} 2-x-2y \, dx\,dy$$

Ex: Evaluate $\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx$

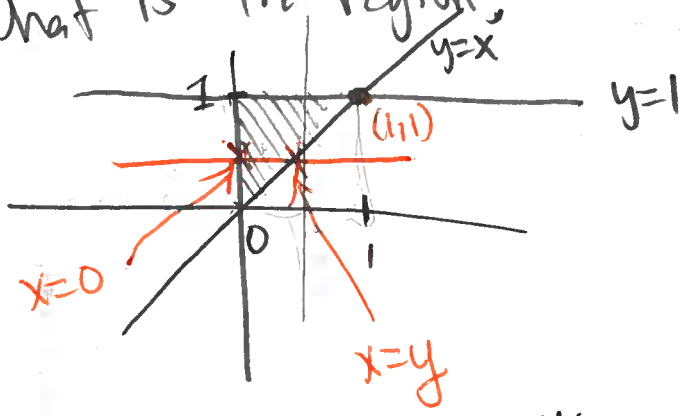
(1) ← exit region on curve $y=1$

(x) ← enter region on curve $y=x$

Soln: Can't do as written.

So try writing as $dx\,dy$.

What is the region?



So,

$$\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx = \int_0^1 \int_0^y \sin(y^2)\,dx\,dy$$

$$= \int_0^1 x\sin(y^2)\Big|_{x=0}^{x=y}\,dy$$

$$= \int_0^1 y\sin(y^2)\,dy$$

$u = y^2$

$du = 2y\,dy$

$\frac{1}{2}du = y\,dy$

$y=0 \to u = 0^2 = 0$

$y=1 \to u = 1^2 = 1$

$$= \frac{1}{2}\int_0^1 \sin(u)\,du = \frac{1}{2}\left[-\cos(u)\Big|_0^1\right.$$

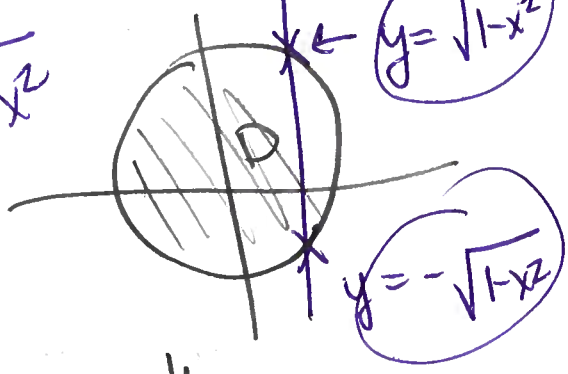
$$= \frac{1}{2}\left[-\cos(1) - (-\cos(0))\right]$$

$$= \frac{1}{2}\left[1 - \cos(1)\right]$$

Trouble w/ circles

$x^2+y^2=1$ unit circle

$y=\pm\sqrt{1-x^2}$



awful to integrate!

⇓

need a different method

⇓

double int in polar coords