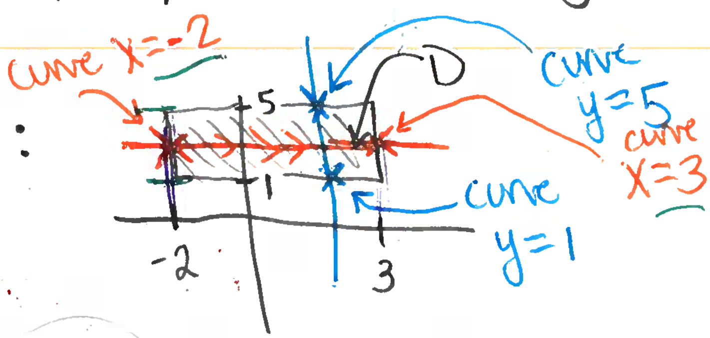
Ex: Let $D = \left\{ (x,y) : \begin{array}{l} -2 \le x \le 3 \\ 1 \le y \le 5 \end{array} \right\}$

Calculate

$$\iint_D x\sin(y) + y\cos(x)\, dA$$

as dydx and as dxdy.

Soln:



as dydx

$$\iint_D x\sin(y) + y\cos(x)\, dA = \int_{-2}^{3} \left[\int_1^5 x\sin(y) + y\cos(x)\, dy \right] dx$$

$$= \int_{-2}^{3} \left(-x\cos(y) + \frac{y^2\cos(x)}{2} \Big|_{y=1}^{y=5} \right) dx$$

$$= \int_{-2}^{3} \left(-x\cos(5) + \frac{25\cos(x)}{2}\right) - \left(-x\cos(1) + \frac{\cos(x)}{2}\right) dx$$

$$= (-\cos(5) + \cos(1)) \int_{-2}^{3} x\, dx + \left(\frac{25}{2} - \frac{1}{2}\right) \int_{-2}^{3} \cos(x)\, dx$$

$$= (-\cos(5) + \cos(1)) \left[\frac{9}{2} - \frac{4}{2}\right] + \left(\frac{24}{2}\right)(\sin(3) - \sin(-2))$$

as dxdy

$x^2\sin(y) - y\ldots$

$$\iint_D x\sin(y)+y\cos(x)\,dA = \int_1^5\left[\int_{-2}^{3} x\sin(y)+y\cos(x)\,dx\right]dy$$

$-\cos(y)$

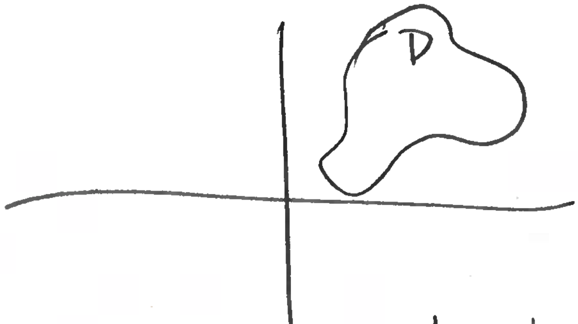
$$= \int_1^5\left[\left(9\sin(y)-y\sin(3)\right)-\left(4\sin(y)+y\sin(-2)\right)\right]dy$$

$$= 5\int_1^5 \sin(y)\,dy + \left(\sin(-2)-\sin(3)\right)\int_1^5 y\,dy \qquad \frac{y^2}{2}$$

$$= 5\left[-\cos(5)+\cos(1)\right] + \underbrace{\left(\sin(-2)-\sin(3)\right)}_{\text{minor mistake — where?}}\left[\frac{25}{2}-\frac{1}{2}\right]$$

More general regions

What to do if D is not a rectangle?



idea: same procedure, but be careful about curves you enter + exit on

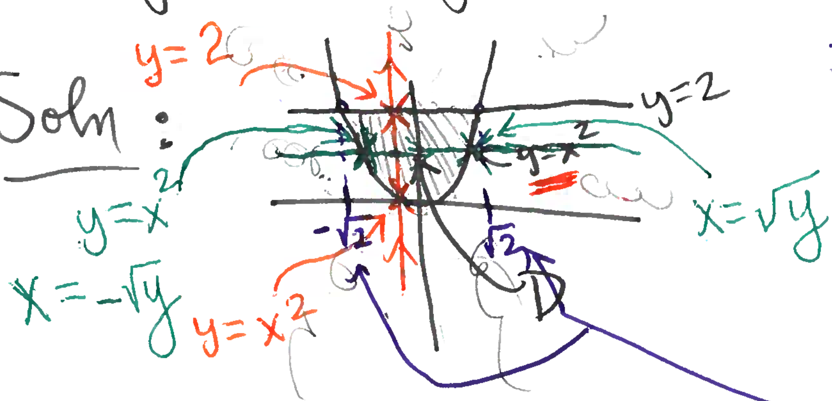
fact: sometimes dydx is easier/harder than dxdy

Ex: Compute $\iint_D xy \, dA$

where D is region bdd by parabola $y=x^2$ and $y=2$.

$y=x^2 \rightarrow x=\pm\sqrt{y}$

Soln:



intersection of $y=x^2$ and $y=2$

$x^2=y=2$

$x^2=2$

$x=\pm\sqrt{2}$

as dydx

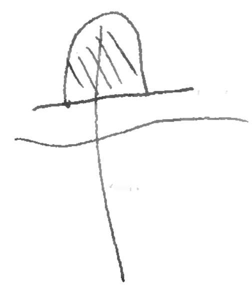
$$\iint_D xy \, dA = \int_{-\sqrt{2}}^{\sqrt{2}} \left[\int_{x^2}^{2} xy \, dy \right] dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left(x \frac{y^2}{2} \Big|_{y=x^2}^{y=2} \right) dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} x\frac{4}{2} - x\frac{x^4}{2} \, dx$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{\sqrt{2}} 4x - x^5 \, dx$$

$$= \frac{1}{2}\left[2x^2 - \frac{x^6}{6}\right|_{-\sqrt{2}}^{\sqrt{2}} = 0$$



as dxdy

$$\iint_D xy\,dA = \int_0^2 \left[\int_{-\sqrt{y}}^{\sqrt{y}} xy\,dx \right] dy$$

$$= \int_0^2 \frac{x^2}{2} y \Big|_{x=-\sqrt{y}}^{x=\sqrt{y}} dy$$

$(-\sqrt{y})^2 = y$

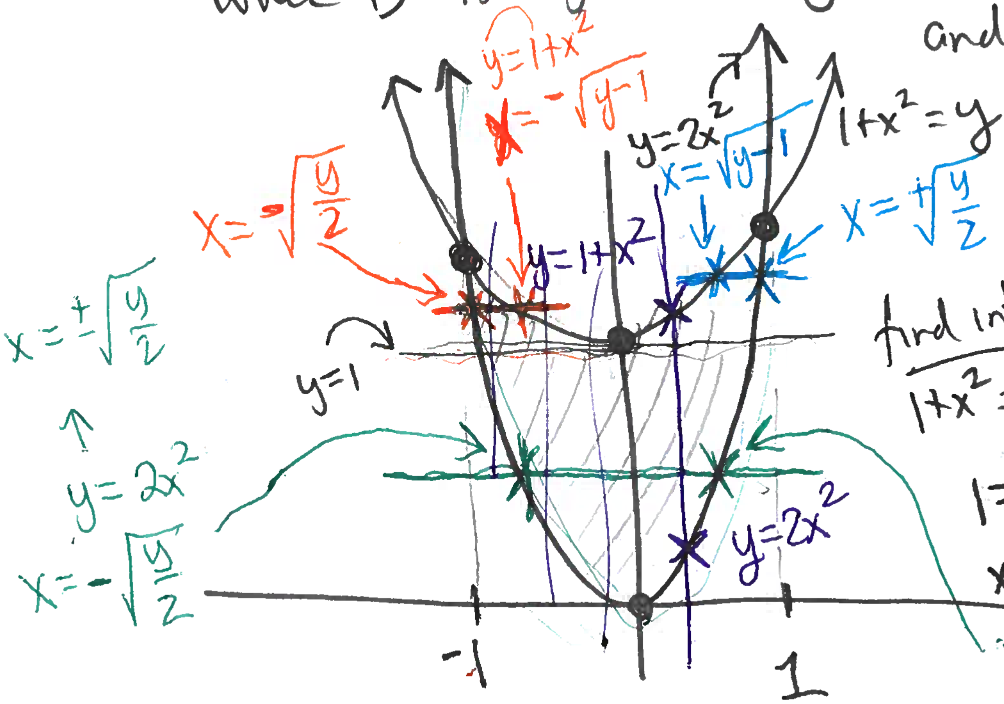
$(\sqrt{y})^2 = y$

$$= \int_0^2 \frac{(\sqrt{y})^2}{2} y - \frac{(-\sqrt{y})^2}{2} y \, dy$$

$$= 0$$

EX: Evaluate $\iint_D 2x+3y\,dA$

where D is region bdd by curves $y=2x^2$ and $y=1+x^2$.



find intersection

$1+x^2 = y = 2x^2$

$1 = x^2$

$x = \pm 1$

as dy dx

$$\iint_D 2x+3y\,dA = \int_{-1}^{1}\int_{2x^2}^{1+x^2} 2x+3y\,dy\,dx$$

as dx dy

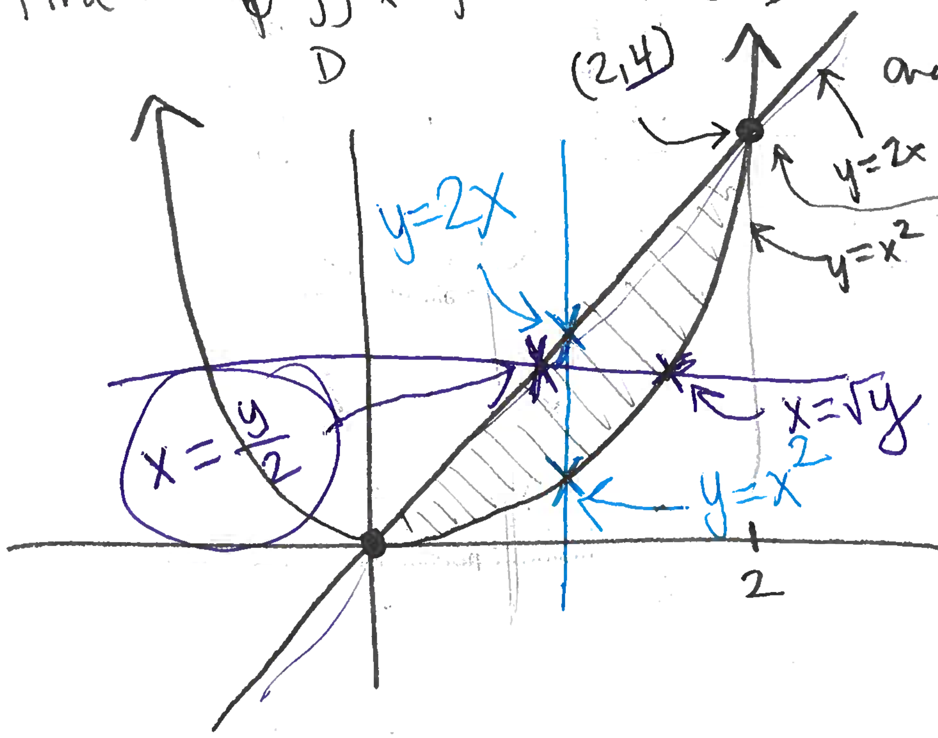
$$\iint_D 2x+3y\,dA = \int_0^1 \int_{-\sqrt{y/2}}^{\sqrt{y/2}} 2x+3y\,dx\,dy$$

$$+\int^{2} \int_{-\sqrt{y/2}}^{-\sqrt{y-1}} 2x+3y\,dx\,dy$$

$$+\int_1^{2} \int_{\sqrt{y-1}}^{\sqrt{y/2}} 2x+3y\,dx\,dy$$

Ex: Find vol of $\iint_D x^2+y^2\,dA$ where D bdd by $y=2x$ and $y=x^2$

$y=x^2 \Rightarrow x=\pm\sqrt{y}$



int pt

$2x=x^2$

$0=x(x-2)$

$x=0$ or $x=2$

as $dy\,dx$

Soln: $$\iint_D x^2+y^2\,dA = \int_1^2 \int_{x^2}^{2x} x^2+y^2\,dy\,dx$$

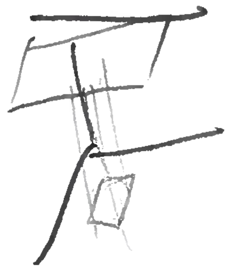
as $dx\,dy$

$$\iint_D x^2+y^2\,dA = \int_0^4 \int_{y/2}^{\sqrt{y}} x^2+y^2\,dx\,dy$$

FACT: Calc 1: $\int_a^b 1\,dx = x\Big|_a^b = b-a$

$= \text{length}((a,b))$

CALC 3: $\iint_D 1\,dA = \text{area}(D)$



$\Downarrow$

can find area formulas from geometry