Revisit box problem from 18 Sep – 21 Sep:

Given $12m^2$ of (no lid) cardboard, maximize volume.

we found earlier: $x=y=2$, $z=1$ (after many pages of work)

Do it as Lagrange multiplier:

$f = xyz$ (vol)

$g = 2xz + 2yz + xy = 12$

$\uparrow$ K

$\nabla f = \langle yz, xz, xy \rangle$

$\nabla g = \langle 2z+y, 2z+x, 2x+2y \rangle$

$$\begin{cases} \langle yz, xz, xy \rangle = \lambda \langle 2z+y, 2z+x, 2x+2y \rangle \\ 2xz+2yz+xy=12 \end{cases}$$

$$\begin{cases} yz = \lambda(2z+y) & \text{(i)} \\ xz = \lambda(2z+x) & \text{(ii)} \\ xy = \lambda(2x+2y) & \text{(iii)} \\ 2xz+2yz+xy=12 & \text{(iv)} \end{cases}$$

Solve (i) for λ: $(2z+y\neq 0)$

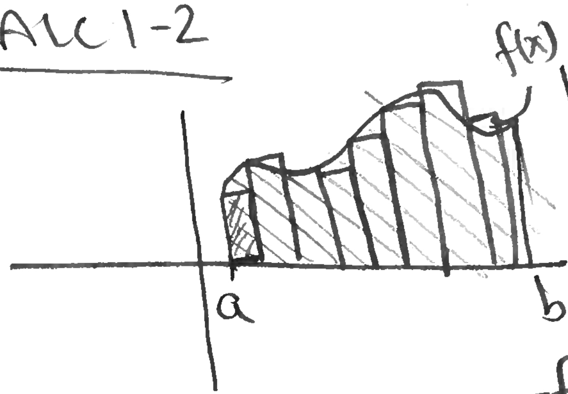
either $2z+y=0$ $\Downarrow$ $yz=0$ $\downarrow$ $y=0$ OR $z=0$ $\xrightarrow{\text{(iv)}}$ $2xz=0$ $\downarrow$ $x=0$ OR $z=0$ $\xrightarrow{\text{(ii)}}$ $0=2\lambda z$ $\downarrow$ $\lambda=0$ OR $z=0$

OR $2z+y\neq 0$ $\downarrow$ $\lambda = \dfrac{yz}{2z+y}$

DO LATER & EMAIL

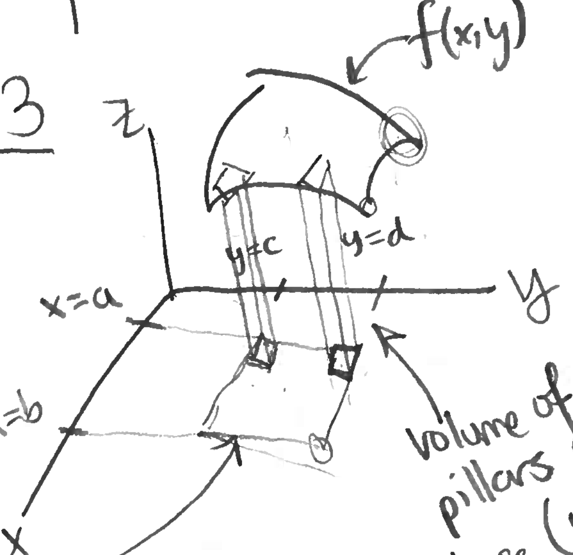
Double integrals

CALC 1-2



lim as rect width → 0 $\int_a^b f(x)\,dx$

CALC 3



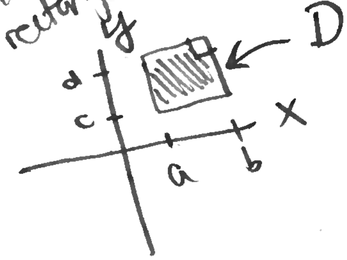
volume of these pillars as their base $(\underbrace{\text{width} * \text{length}}_{\text{Area}}) \to 0$

$\Downarrow$

This gives us the double integral

$$\iint_D f(x,y)\,dA$$

this rectangle is



CALC1 → $\int$ = area under curve

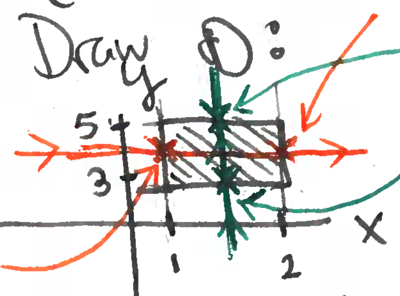
CALC2 → $\iint$ = vol under surface

$\iint$ over rectangular regions (easy!)

Ex: Compute $\iint_D xy\, dA$ where D is region

$$D = \{(x,y) : 1 \le x \le 2,\ 3 \le y \le 5\}$$

Soln: Draw D:



We can write $\iint_D xy\, dA$ either as

dy dx – type or dx dy – type

"shoot the arrow" in y-dir

"shoot the arrow" in x-dir

$$\int y\, dy = \frac{y^2}{2} + C$$

dy dx – type

$$\iint_D xy\, dA = \int_1^2 \int_3^5 xy\, dy\, dx = \int_1^2 \underbrace{\left[\int_3^5 xy\, dy\right]}_{\text{calculate first}} dx$$

5 ← exit curve

3 ← entrance curve

$$= \int_1^2 x\frac{y^2}{2}\Big|_{y=3}^{y=5} dx$$

$$= \frac{1}{2}\int_1^2 25x - 9x\, dx = \frac{16}{2}\int_1^2 x\, dx$$

$$= 8\left(\frac{2^2}{2} - \frac{1}{2}\right)$$

$$= 4 \cdot 3 = 12$$

dxdy-type

$$\iint_D xy\,dA = \int_3^5 \left[\int_1^2 xy\,dx\right] dy$$

$$= \int_3^5 \frac{x^2}{2}y \Big|_{x=1}^{x=2} dy$$

$$= \frac{1}{2}\int_3^5 2^2y - 1^2y\,dy$$

$$= \frac{1}{2}\int_3^5 3y\,dy$$

$$= \frac{3}{2}\,\frac{y^2}{2}\Big|_{y=3}^{y=5} = \frac{3}{4}[25-9]$$

$$= \frac{3}{\cancel{4}}[16] = 3\cdot 4 = 12$$