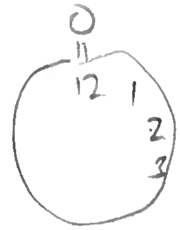
Continued from 18 Sept:

$$Vol_y = -4x^3y - 2x^2y^2 + 24x^2 \overset{\text{set}}{=} 0 \quad (i)$$

$$Vol_x = -2x^2y^2 - 4xy^3 + 24y^2 = 0 \quad (ii)$$



$$(i) \rightarrow 2x^2(-2xy - y^2 + 12) = 0$$

$(a+b)^2 = a^2 + 2ab + b^2$

$2x^2 = 0$

$\downarrow$

$x = 0$

$\Downarrow$ (ii)

$24y^2 = 0$

$\downarrow$

$y = 0$

$\Downarrow$

$(0,0)$ is a crit pt

$-2xy - y^2 + 12 = 0$

$\downarrow$ solve for x

$$\boxed{x = \frac{y^2 - 12}{-2y} = -\frac{y}{2} + \frac{6}{y}}$$

$\Downarrow$ plug into (ii)

$$-2\left(-\frac{y}{2} + \frac{6}{y}\right)^2 y^2 - 4\left(-\frac{y}{2} + \frac{6}{y}\right)y^3 + 24y^2 = 0$$

$$2\left(-\left[\frac{y^2}{4} - 6 + \frac{36}{y^2}\right]y^2 - 2\left(-\frac{y}{2} + \frac{6}{y}\right)y^3 + 12y^2\right) = 0$$

$$-\frac{1}{4}y^4 + 6y^2 - 36 + y^4 - \cancel{12y^2} + \cancel{12y^2} = 0$$

$$\frac{3}{4}y^4 + 6y^2 - 36 = 0$$

$$\frac{1}{4}y^4 + 2y^2 - 12 = 0$$

$$y^4 + 8y^2 - 48 = 0$$

$\|$

let $w = y^2$

$w^2 + 8w - 48 = 0$

$(w+12)(w-4) = 0$

$\downarrow$

$w = -12, \quad w = 4$

$y^2 = -12, \quad y^2 = 4$

$y = \pm\sqrt{-12}$ $\downarrow$ complex roots $\downarrow$ NOT for here

$y = \pm 2$

$y = +2$: $x = -\frac{2}{2} + \frac{6}{2} = -1 + 3 = 2 \Rightarrow (2,2)$

$y = -2$: $x = \frac{2}{2} + \frac{6}{-2} = -2 \Rightarrow (-2,-2)$

We found 3 crit pts:

$(0,0), (2,2), (-2,-2)$

NOT PHYSICALLY MEANINGFUL

ignore!

At $x=y=2$:

$$D = \langle \text{from computer} \rangle$$

$$= -x^2 + \frac{2(x^6+18x^4)}{(x+y)^4} - \frac{6(x^5+12x^3)}{(x+y)^3} + \frac{5x^4+36x^2}{(x+y)^2}$$

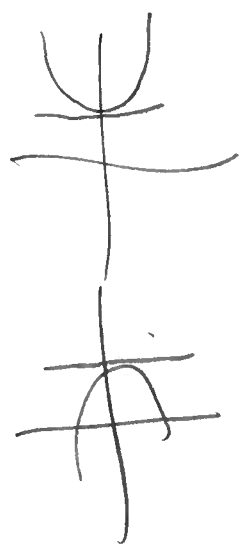
at $x=y=2$:

$$D = -4 + \frac{2(2^6+18\cdot 2^{14})}{4^4} - \frac{6(2^5+12\cdot 2^3)}{4^3} + \frac{5\cdot 2^4+36\cdot 4}{4^2}$$

$$>0$$

and $f_{xx} = -\dfrac{y^2(y^2+12)}{(x+y)^3}$

so $f_{xx}(2,2) = \dfrac{-4(4+12)}{4^3} < 0$



$\Rightarrow$ local max at $(2,2)$

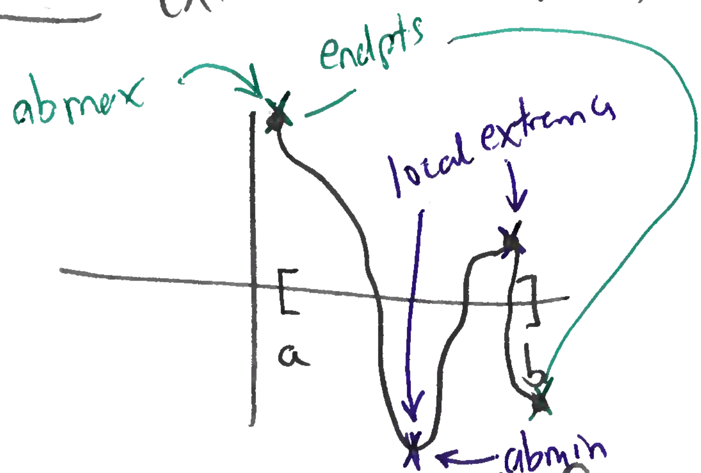
b/c $D>0$

$f_{xx}<0$

$$\Rightarrow z = \frac{12-xy}{2x+2y} \Rightarrow z\Big|_{(2,2)} = \frac{12-4}{8} = 1$$

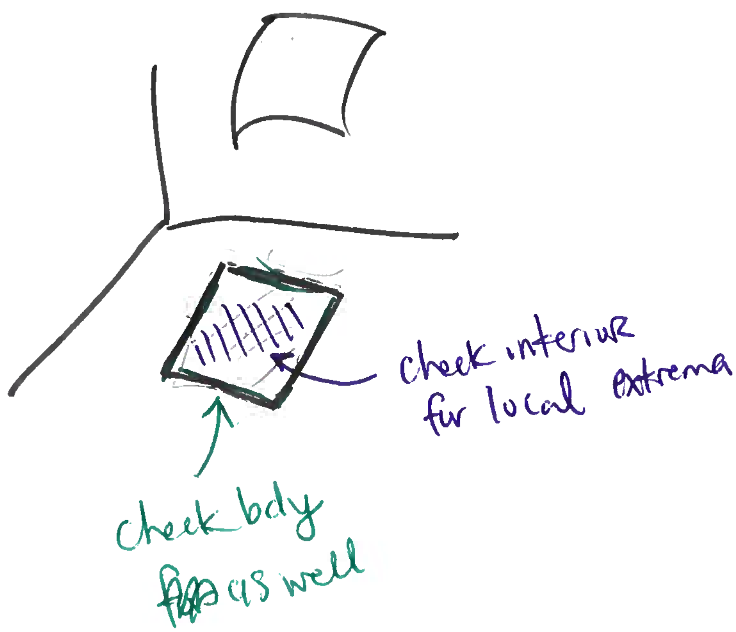
$\boxed{\text{So max volume comes with square base w/ side length 2 + vertical length 1}}$

CALC I extreme value thm



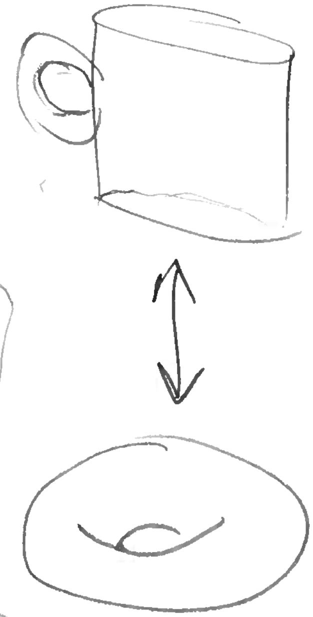
on [a,b] could find absolute max by finding all crit pts + checking endpoints

CALC3



Some topological notions

We need a calc3 version of closed interval $[a,b]$.



(a,b) - open

$\left.\begin{array}{l}[a,b)\\(a,b]\end{array}\right\}$ clopen

* $[a,b]$ is called <u>closed</u> because it contains its boundary pts

* $[a,b]$ is <u>bounded</u> meaning it fits inside some sufficiently large "disk" in $\mathbb{R}^1$

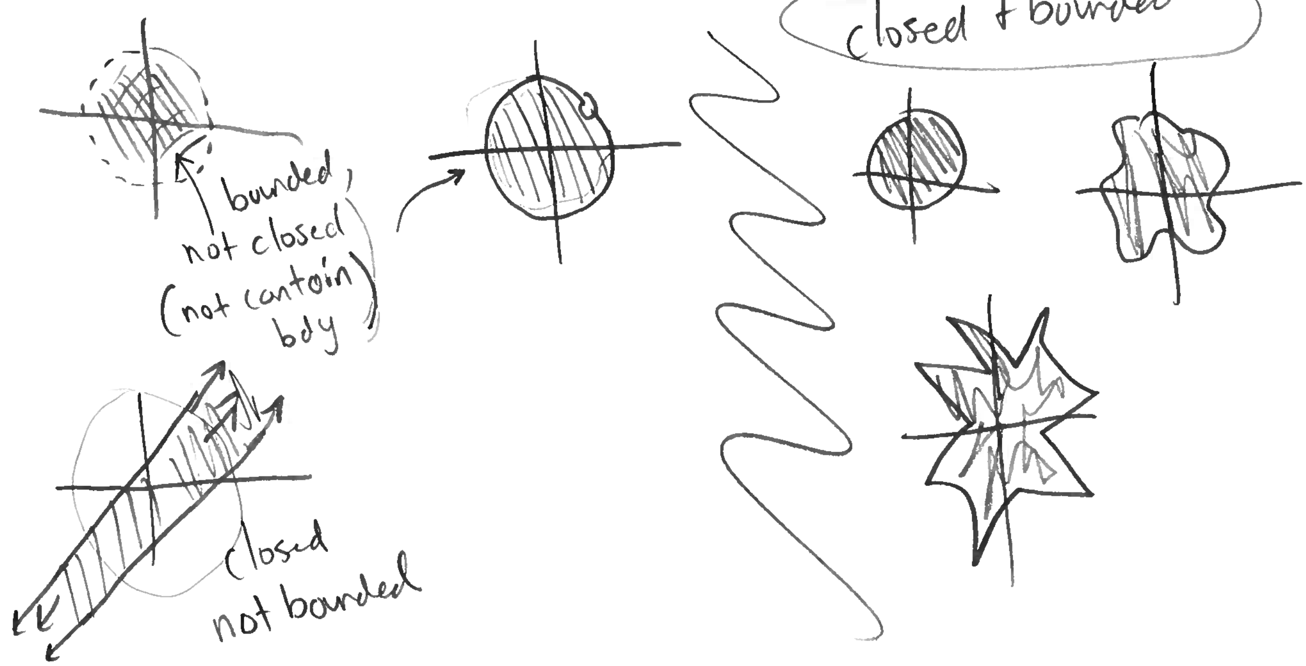
int

Closed + bounded sets in $\mathbb{R}^2$

"compact set"

↓

closed + bounded



Extreme Value Thm (Calc 3)

If f continuous on a closed + bdd set D (in domain), then f attains its abmax + abmin at some values in D.

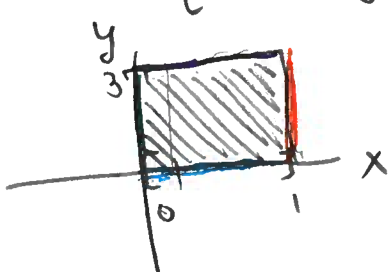
STRATEGY TO FIND

1. find value of f at crit pts
2. find all extreme values on bdy
3. largest $\to$ abmax
   smallest $\to$ abmin

Ex: Find abmin + abmax of $f(x,y) = y^2 + xy - 4x^2$

on $D = \left\{(x,y) : \begin{array}{l} 0 \le x \le 1 \\ 0 \le y \le 3 \end{array}\right\}$

Soln:



Find cp's

$f_x = y - 8x \overset{\text{set}}{=} 0 \quad \text{(i)} \longrightarrow y = 8x$

$f_y = 2y + x \overset{\text{set}}{=} 0 \quad \text{(ii)} \longrightarrow 16x + x = 0$

$x = 0 \longrightarrow y = 0$

$\Rightarrow \boxed{(0,0) \text{ is a c.p.}}$

7b

Our boundary has 4 sides:

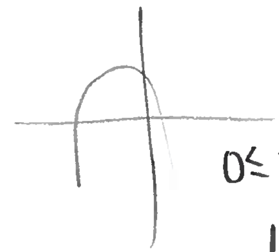
TOP ~ $0 \le x \le 1$, $y = 3$

LEFT - $0 \le y \le 3$, $x = 0$

BOTTOM - $0 \le x \le 1$, $y = 0$

RIGHT - $0 \le y \le 3$, $x = 1$

Top: $y = 3 \rightarrow f(x,3) = 9 + 3x - 4x^2$



$\downarrow$ by CALC I

$$\frac{d}{dx} f(x,3) = 3 - 8x \overset{\text{set}}{=} 0$$

$$x = \frac{3}{8}$$

$0 \le \frac{3}{8} \le 1$ ✓

$\Longrightarrow$ CALC I 2nd deriv test

$$\frac{d^2}{dx^2} f(x,3) = -8$$

$\boxed{\text{at } x = \tfrac{3}{8}\text{, top has a max there}}$