Ex: Find + classify crit pts of

$$f(x,y) = x^2 + 4xy + 2y^2 + 4x - 8y + 3$$

Soln: $f_x = 2x + 4y + 4 \overset{\text{set}}{=} 0$ (i)

$f_y = 4x + 4y - 8 \overset{\text{set}}{=} 0$ (ii)

Solve (i) for x:

$$\boxed{x = -2 - 2y}$$

Plug into (ii):

$$4(-2-2y) + 4y - 8 = 0$$

$$-8 - 8y + 4y - 8 = 0$$

$$-4y = 16$$

$$\boxed{y = -4}$$

$$x = -2 - 2(-4) = 6$$

$\Rightarrow (6, -4)$

Classify $(6,-4)$

$f_{xx} = 2$, $f_{xy} = 4 = f_{yx}$, $f_{yy} = 4$

$$D = f_{xx}f_{yy} - f_{xy}f_{yx} = 8 - 16 = -8 < 0$$

$\Rightarrow$ saddle pt at $(6,-4)$

<u>Ex</u>: Find shortest distance from point $(1,2,3)$ to plane

$$2x+3y-z=7.$$

<u>Soln</u>: Recall: distance in $\mathbb{R}^3$

$$d\big((a,b,c),(d,e,f)\big)=\sqrt{(a-d)^2+(b-e)^2+(c-f)^2}$$

Rewrite plane eqt as

$y=x^2$

$$z=\underbrace{2x+3y-7}_{f(x,y)}$$

Point on plane:

$$(x,y,2x+3y-7)$$

So, we need to optimize

$$d\big((1,2,3),(x,y,2x+3y-7)\big)=\sqrt{(1-x)^2+(2-y)^2+(3-2x-3y+7)^2}$$

<u>TRICK</u>: it is sufficient to minimize the square of the distance

So, we need to minimize

$$g(x,y)=(1-x)^2+(2-y)^2+(10-2x-3y)^2$$

$$\overset{-2+2x}{}\qquad\qquad -40+8x+12y$$

$$g_x=2(1-x)(-1)+2(10-2x-3y)(-2)$$

$$\rightarrow\ =-42+10x+12y\overset{\text{set}}{=}0$$

$$g_y=2(2-y)+2(10-2x-3y)(-3)$$

$$=4-2y-60+12x+18y\overset{\text{set}}{=}0$$

③

$$\begin{cases} g_x = -42 + 10x + 12y \overset{\text{set}}{=} 0 & \text{(i)} \\ g_y = -56 + 16y + 12x \overset{\text{set}}{=} 0 & \text{(ii)} \end{cases}$$

(i) $\rightarrow$ $\boxed{x = \frac{38}{6} - 2y} = \frac{19}{3} - 2y$

$\Downarrow$ into (ii)

$3 \cdot 19$
$\times 4$
76

$$-56 + 16y + 12\left(\frac{19}{3} - 2y\right) = 0$$

$$-56 + 16y + 76 - 24y = 0$$

$$20 - 8y = 0$$

$\leftarrow$ WRONG

$$\boxed{y = \frac{20}{8}}$$

$$x = \frac{19}{3} - 2\left(\frac{20}{8}\right)$$

$$= \frac{19}{3} - 5 = \frac{4}{3} \not\Rightarrow \left(0, \frac{1}{2}\right)$$

computer

$$g_{xx} = 10, \quad g_{xy} = 12 = g_{yx}, \quad g_{yy} = 16$$

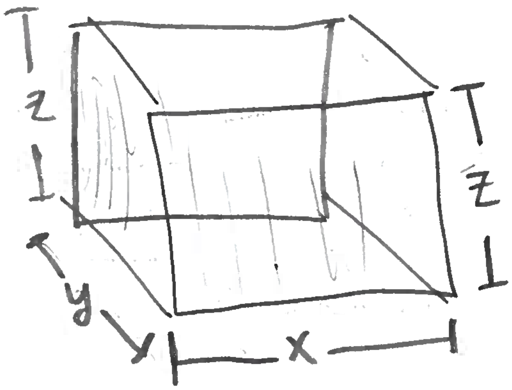
$$\Rightarrow D = g_{xx}g_{yy} - g_{xy}g_{yx} = 10(16) - 12^2 > 0$$

and $g_{xx} > 0$

$\Rightarrow$ minimum! $\longleftarrow$ picture seems still a problem

Ex: Construct box, no lid, from $12m^2$ of cardboard.
Find max volume.

Soln:



Total area:

constraint

$$\boxed{2xz + 2yz + xy = 12}$$

$2xz$ ← front + back

$2yz$ ← left + right side

xy ← bottom

$$Vol = xyz$$

Use constraint to replace z in vol equation.

constraint: $z(2x+2y)+xy=12$

$$z = \frac{12-xy}{2x+2y}$$

Therefore,

$$Vol = xyz = xy\left(\frac{12-xy}{2x+2y}\right) = \frac{12xy - x^2y^2}{2x+2y}$$

$$Vol_x = \frac{(2x+2y)(12y-2xy^2) - (12xy - x^2y^2)(2)}{(2x+2y)^2} \overset{set}{=} 0$$

$$Vol_y = \frac{(2x+2y)(12x-2x^2y) - (12xy - x^2y^2)(2)}{(2x+2y)^2} = 0$$

Mult denoms off

$$Vol_y = -4x^3y - 2x^2y^2 + 24x^2 \overset{set}{=} 0 \quad (i)$$

$$Vol_x = -2x^2y^2 - 4xy^3 + 24y^2 \overset{set}{=} 0 \quad (ii)$$

$$(i) \rightarrow 2x^2(-2xy - y^2 + 12) = 0$$

$2x^2 = 0$ $\qquad$ $-2xy - y^2 + 12 = 0$

$x = 0$

$\downarrow$ plug (ii)

$24y^2 = 0$

$y = 0$

$\downarrow$

$(x, y) = (0, 0)$