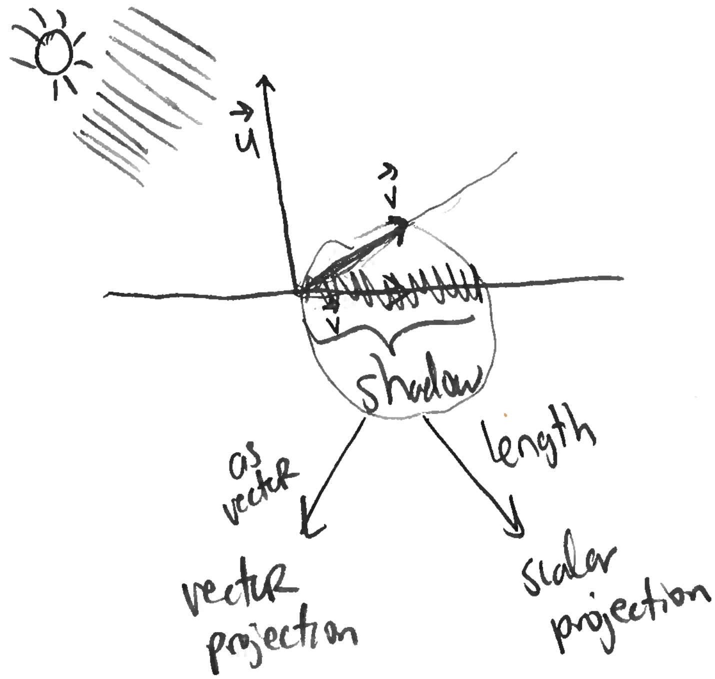
Vector and scaler projections



vector projection $\rightarrow$ $\text{proj}_{\vec v}(\vec u) = \left(\frac{\vec u \cdot \vec v}{\vec v \cdot \vec v}\right)\vec v = \left(\frac{\vec u \cdot \vec v}{\|\vec v\|^2}\right)\vec v$

scalar proj $\rightarrow$ $\text{comp}_{\vec v}(\vec u) = \frac{\vec u \cdot \vec v}{\|\vec v\|}$

<u>Notice:</u> $\|\text{proj}_{\vec v}(\vec u)\| = \left\| \frac{\vec u \cdot \vec v}{\|\vec v\|^2}\vec v \right\| = \left| \frac{\vec u \cdot \vec v}{\|\vec v\|^{\cancel{2}}} \right| \cancel{\|\vec v\|} = |\text{comp}_{\vec v}(\vec u)|$

<u>EX</u>: If $\vec u = \langle 1,2,3 \rangle$ and $\vec v = \langle 1,-1,1 \rangle$

then

$$\text{proj}_{\vec v}(\vec u) = \frac{\langle 1,2,3\rangle \cdot \langle 1,-1,1\rangle}{(1^2+(-1)^2+1^2)}\langle 1,-1,1\rangle$$

$$= \frac{1-2+3}{3}\langle 1,-1,1\rangle = \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle$$

2/3

This is used in linear algebra to take a finite set of vectors & create ~~an or~~ a mutually orthogonal set of vectors. – Gram-Schmidt $\{1, x, x^2, \ldots\}$

~ Applies to – quantum mechanics (end up with $\underbrace{\text{Hermite polynomials}}$)

Chain Rule

CALC I

$$\frac{d}{dx}\left[(2x+1)^3\right] = \frac{d}{dx}\left[f(g(x))\right]$$

$$= f'(g(x))\, g'(x)$$

$f(x) = x^3$

$g(x) = 2x+1$

$\frac{dg}{dx} = 2$

$$\boxed{\frac{df}{dx} = \frac{df}{\cancel{dg}} \frac{\cancel{dg}}{dx}} = 3(2x+1)^2 \cdot 2$$

$$\frac{d}{d\xi} \xi^n = n \xi^{n-1}$$

$$\frac{d}{d\underset{g}{(2x+1)}} \underset{g^3}{(2x+1)^3} = 3(2x+1)^2 \cdot 2$$

Calc 3

one var

if $x = x(t)$, $y = y(t)$, $z = f(x,y)$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Ex: $f(x,y) = x^2 + xy$

$x(t) = t^2 + 7$ $\quad y(t) = \sin(t)$

$$\frac{df}{dt} = \frac{d}{dt}\left[(t^2+7)^2 + (t^2+7)\sin(t)\right]$$

$$= 2(t^2+7)(2t) + \left[2t\sin(t) + (t^2+7)\cos(t)\right]$$

but also

$$\frac{df}{dt} = (2x+y)(2t) + (x)(\cos(t))$$

$$= \left[2(t^2+7) + \sin(t)\right](2t) + (t^2+7)\cos(t)$$

if $x = x(s,t)$, $y = y(s,t)$, $z = f(x,y)$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Ex: if $f(x,y) = x^2 y + e^{xy}$

and $x = s^2 t^2$ $\quad y = s\cos(t)$

Then,

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$

$$= \left(2xy + ye^{xy}\right)\left(2st^2\right) + \left(x^2 + xe^{xy}\right)\left(\cos(t)\right)$$

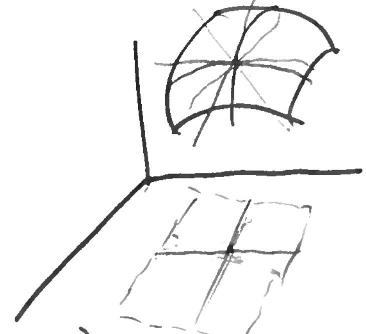
$$= \left(2s^3t^2\cos(t) + s\cos(t)e^{s^3t^2\cos(t)}\right)\left(2st^2\right)$$

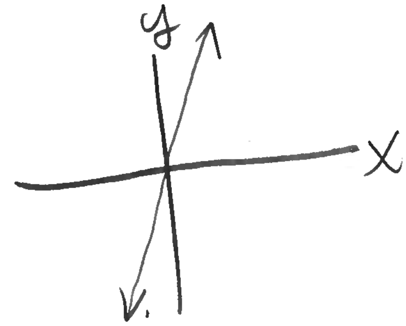
$$+ \left(s^4t^4 + s^2t^2e^{s^3t^2\cos(t)}\right)\cos(t)$$

$$\frac{\partial f}{\partial t} = \left(2xy + ye^{xy}\right)2s^2t + \left(x^2 + xe^{xy}\right)\left(-s\sin(t)\right)$$

$=$ (sub in what $x=$ and what $y=$)

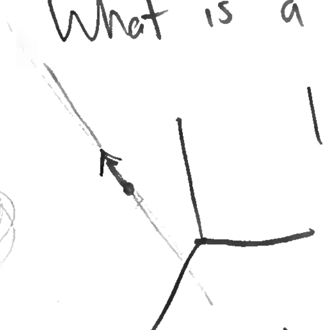
Directional derivatives





A directional derivative allows us to take a partial derivative in a direction that is some combo of x-direction + y-direction.

What is a direction?



$\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\langle 1,1 \rangle$

$\langle 2,2 \rangle$

lines → needed a point
parallel vector ← encoded direction
$P + t\vec{v}$

Here — direction is also given by a vector, BUT it <u>must</u> be a unit vector.

[* Because a <u>unit</u> vector represents a combination that won't change value of derivative.] *

If $\vec{u} = \langle a, b \rangle$ is a unit vector $(\|\vec{u}\| = 1)$, then the directional derivative of $f(x,y)$ at (x,y) in direction $\vec{u}$ is given by

$$D_{\vec{u}} f(x,y) = \underbrace{a f_x(x,y) + b f_y(x,y)}_{\text{"linear combination of } \frac{\partial f}{\partial x} \text{ and } \frac{\partial f}{\partial y}\text{"}}$$

Ex: Find $D_{\vec{u}}f$ at $(1,2)$ of $f(x,y)=x^2y$

and $\vec{u} = \langle \underset{a}{\frac{1}{\sqrt{2}}}, \underset{b}{\frac{1}{\sqrt{2}}} \rangle$.

Soln: $\|\vec{u}\| = \sqrt{\frac{1}{2}+\frac{1}{2}} = \sqrt{1} = 1$ ✓

$f_x = 2xy \quad f_y = x^2$

$$D_{\vec{u}}f(1,2) = \left(\frac{1}{\sqrt{2}}\right)f_x(1,2) + \left(\frac{1}{\sqrt{2}}\right)f_y(1,2)$$

$$= \left(\frac{1}{\sqrt{2}}\right)4 + \left(\frac{1}{\sqrt{2}}\right)1$$

$$= \frac{5}{\sqrt{2}}$$