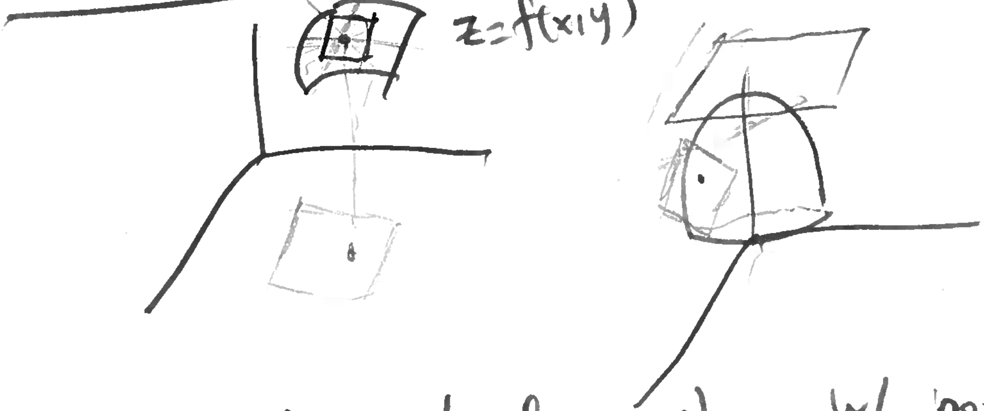
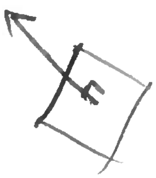
Partial derivatives





<u>Recall</u>: eqt of a plane w/ point $P=(x_0,y_0,z_0)$ and a normal vector $\vec{n}=\langle a,b,c\rangle$ is

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

<u>Ex</u>: Find tangent plane to

$$z=2x^2+y^2$$

at $(1,1,3)$.

<u>Soln</u>: $f(x,y)=2x^2+y^2$

$$\frac{\partial f}{\partial x}=4x \longrightarrow \left.\frac{\partial f}{\partial x}\right|_{(1,1)}=4 \rightsquigarrow a$$

$$\frac{\partial f}{\partial y}=2y \longrightarrow \left.\frac{\partial f}{\partial y}\right|_{(1,1)}=2 \rightsquigarrow b$$

For $z=f(x,y) \rightsquigarrow c=-1$

$P=(1,1,3)$, $\vec{n}=\langle 4,2,1\rangle$

$$4(x-1)+2(y-1)-(z-3)=0 \rightarrow \begin{aligned} z&=4x-4+2y-2+3\\ &=4x+2y-3\end{aligned}$$

Cont:

Find tan plane at $(0,0,0)$

$$\left.\frac{\partial f}{\partial x}\right|_{(0,0)} = 0 \qquad \Rightarrow \quad \vec{n} = \langle 0,0,-1\rangle$$

$$\left.\frac{\partial f}{\partial y}\right|_{(0,0)} = 0$$

tan plane:

$$0(x-0)+0(y-0)-(z-0)=0$$

$$z=0$$

EX: Find tan plane of hyperboloid in 2 sheets

$$x^2-y^2-z^2=4$$

at $(3,1,z_0)$ and at $\cancel{(-2,2,z_1)}$

$\cancel{(-2)^2-2^2-z_1^2=4}$

$\cancel{z_1^2=-4}$

Soln:

$$z^2 = x^2-y^2-4$$

$$z = \pm\sqrt{x^2-y^2-4} = \underbrace{\pm\left(x^2-y^2-4\right)^{1/2}}_{f(x,y)}$$

$9-1-z_0^2=4$

$z_0^2=4$

$z_0=\pm 2$

$z_0=+2$

$(3,1,2)$

$(3,1,-2)$

9-1-4

$$\frac{\partial f}{\partial x} = \pm\frac{1}{2}\left(x^2-y^2-4\right)^{-1/2}(2x) \longrightarrow \left.\frac{\partial f}{\partial x}\right|_{(3,1)} = \pm\frac{1}{2}4^{-1/2}(6) = \pm\frac{6}{4} = \pm\frac{3}{2}$$

$$\frac{\partial f}{\partial y} = \pm\frac{1}{2}\left(x^2-y^2-4\right)^{-1/2}(-2y) \longrightarrow \left.\frac{\partial f}{\partial y}\right|_{(3,1)} = \pm\frac{1}{2}\left(4^{-1/2}\right)(-2) = \pm\left(\frac{-2}{4}\right) = \mp\frac{1}{2}$$

Tan planes

$P=(3,1,2)$

$\vec{n}=\langle \frac{3}{2}, -\frac{1}{2}, -1\rangle$

$\downarrow$

$\frac{3}{2}(x-3)-\frac{1}{2}(y-1)-(z-2)=0$

$P=(3,1,-2)$

$\vec{n}=\langle -\frac{3}{2}, \frac{1}{2}, -1\rangle$

$\downarrow$

$-\frac{3}{2}(x-3)+\frac{1}{2}(y-1)-(z+2)=0$

Ex: Find tan plane to

$$f(x,y)= x^2\sin(y^2)$$

at $(1,\sqrt{\pi},0)$

$\sin((\sqrt{\pi})^2)=\sin(\pi)=0$

Soln: $\frac{\partial f}{\partial x}=2x\sin(y^2) \longrightarrow \left.\frac{\partial f}{\partial x}\right|_{(1,\sqrt{\pi})}=0$

$\frac{\partial f}{\partial y}=x^2\cos(y^2)(2y) \longrightarrow \left.\frac{\partial f}{\partial y}\right|_{(1,\sqrt{\pi})}=(1^2)(-1)(2\sqrt{\pi})$

$\cos((\sqrt{\pi})^2)=\cos(\pi)=-1$

$=-2\sqrt{\pi}$

$\vec{n}=\langle 0,-2\sqrt{\pi},-1\rangle$

$0(x-1)+(-2\sqrt{\pi})(y-\sqrt{\pi})-(z-0)=0$

$$\boxed{z=-2\sqrt{\pi}\,y+2\pi}$$

✓ f_{xx}

f_{xy}

f_{yx}

✓ f_{yy}

Ex: $f(x,y) = 2\sin(x-y) + \cos(x+y)$

Find all 2^{nd} partial derivatives of f.

$$\frac{\partial f}{\partial x} \equiv f_x = 2\cos(x-y) - \sin(x+y)$$

$$\frac{\partial f}{\partial y} \equiv f_y = \boxed{-2\cos(x-y) - \sin(x+y)}$$

$$\frac{\partial^2 f}{\partial x^2} \equiv f_{xx} = -2\sin(x-y) - \cos(x+y)$$

$$\frac{\partial^2 f}{\partial y^2} = f_{yy} = +\sin(x-y)(-1) - \cos(x+y)$$

$$\frac{\partial^2 f}{\partial y \partial x} \equiv f_{xy} = +2\sin(x-y) - \cos(x+y)$$

equal

$$\frac{\partial}{\partial y}\left[\frac{\partial}{\partial x} f\right]$$

$$\frac{\partial^2 f}{\partial x \partial y} \equiv f_{yx} = +2\sin(x-y) - \cos(x+y)$$