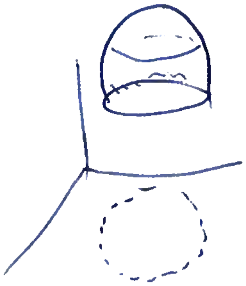
Recall Polar coordinates

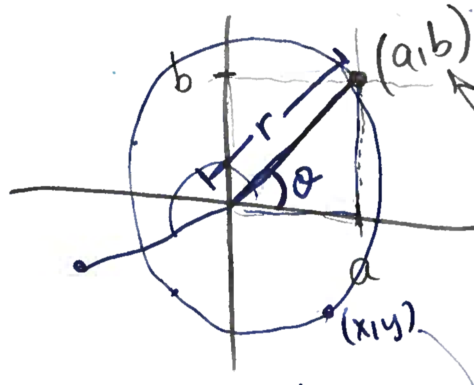
just TRIG



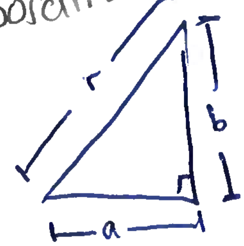
radius angle

$(a,b) = (r, \theta)$

$r > 0$ $\quad 0 \le \theta \le 2\pi$



Rectangular coordinates



circle of radius r

$\Updownarrow$

$x^2 + y^2 = r^2$

$\Updownarrow$

<u>MORAL:</u> any time "x^2+y^2" or similar appears

$\Downarrow$

good idea to consider polar

EX: $$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$$

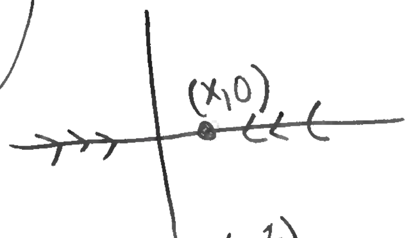
$$\boxed{\text{CALC 1} \quad \lim_{x\to 0}\frac{\sin(x)}{x}=1}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$\frac{\sin(x)}{x} = \frac{\rule{2em}{0.4pt}}{x}$$

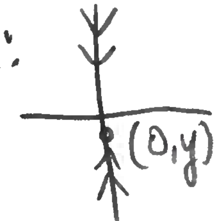
$$= 1 - \underbrace{\frac{x^2}{3!}}_{\to 0} + \underbrace{\frac{x^4}{5!}}_{\to 0} - \underbrace{\frac{x^6}{7!}}_{\to 0} + \cdots$$

Path 1



$$\lim_{(x,0)\to(0,0)} \frac{\sin(x^2)}{x^2} = 1$$

Path 2:



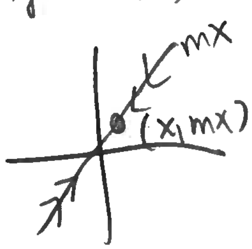
$$\lim_{(0,y)\to(0,0)} \frac{\sin(y^2)}{y^2} = 1$$

$$\lim_{x\to 0}\frac{\sin(5x)}{x}$$

$$= \lim_{x\to 0}\frac{\sin(5x)}{x}\left(\frac{5}{5}\right)$$

$$= 5\underbrace{\lim_{x\to 0}\frac{\sin(5x)}{5x}}_{=1} = 5$$

Path 3:



$$\lim_{(x,mx)\to(0,0)} \frac{\sin(x^2+m^2x^2)}{x^2+m^2x^2} = 1$$

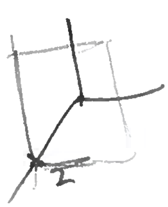
$\uparrow$ $y=mx$

Polar:

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} \underset{\text{polar}}{=} \lim_{r\to 0}\frac{\sin(r^2)}{r^2} = 1$$

independent of θ

shows the limit DOES EXIST!!



Most of time — can plug in the value for a limit:

<u>Ex</u>: $\lim_{(x,y)\to(7,-1)} e^{xy\sin(e^{xy})}$

$= e^{-7\sin(e^{-7})}$

<u>Recall</u>:

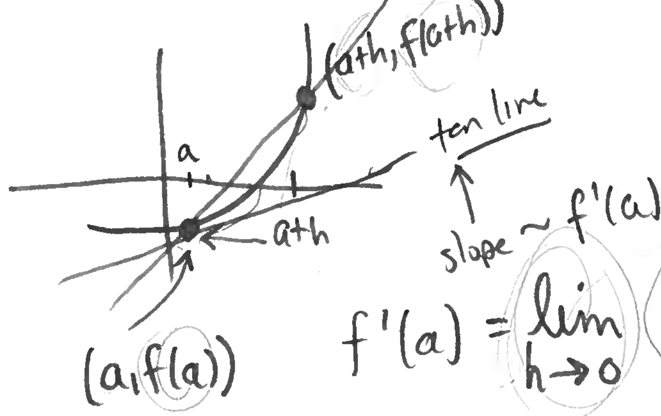
$\checkmark\, f: \mathbb{R}\to\mathbb{R}$ calc 1

$\checkmark\, f: \mathbb{R}\to\mathbb{R}^n$ space curve ~~calculus~~

$f: \mathbb{R}^n\to\mathbb{R}$ partial derivs/dbl $\iint$ etc

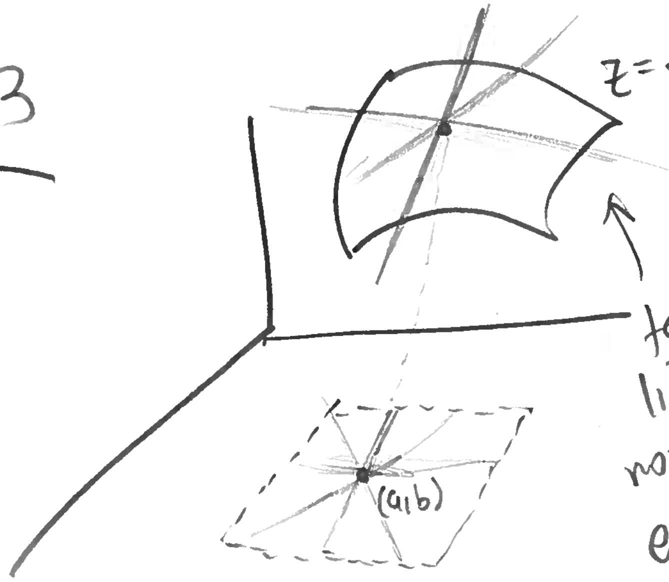
$f: \mathbb{R}^n\to\mathbb{R}^m$ vector calc

<u>Recall</u>:



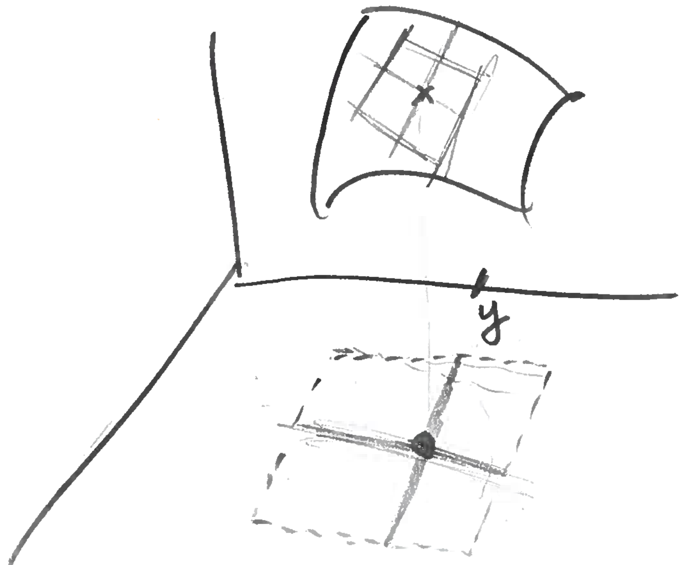
$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

Calc 3



$z = f(x,y)$

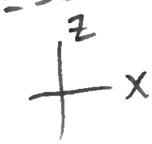
tangent lines are not "powerful" enough here



$y = k$

$\Updownarrow$

$z = \_\_x\_\_$



$\cancel{\frac{\partial f}{\partial x}}$ $\cancel{\frac{\partial f}{\partial y}}$

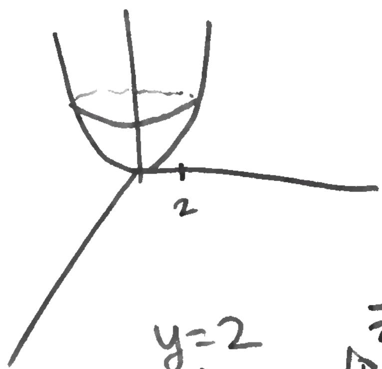
Two derivatives here: \partial

"x-derivative" — $\boxed{\dfrac{\partial f}{\partial x} = \lim\limits_{h \to 0} \dfrac{f(x+h,y) - f(x,y)}{h}}$

"y-derivative" — $\boxed{\dfrac{\partial f}{\partial y} = \lim\limits_{h \to 0} \dfrac{f(x,y+h) - f(x,y)}{h}}$

$z = x^2+y^2$

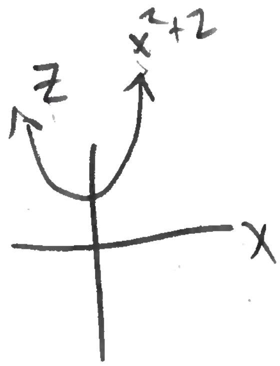
Ex: $f(x,y) = x^2+y^2$



$y=2$

$\downarrow$

$z = x^2+2$



$\longrightarrow \quad \frac{dz}{\partial x} = 2x$

$$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial}{\partial x}[x^2+y^2] = \frac{\partial}{\partial x}[x^2] + \cancel{\frac{\partial}{\partial x}[y^2]}^{\,0}$$

calc 1

$= 2x$

y is thought of by $\frac{\partial}{\partial x}$ as a constant

Ex: $\frac{\partial}{\partial x}[x^2y+yx^3] = \frac{\partial}{\partial x}[x^2y] + \frac{\partial}{\partial x}[yx^3]$

$$= y\underbrace{\frac{\partial}{\partial x}[x^2]}\, + y\frac{\partial}{\partial x}[x^3]$$

$$= 2xy + 3x^2y$$

$$\frac{\partial}{\partial y}[x^2y+yx^3] = x^2\frac{\partial}{\partial y}[y] + x^3\frac{\partial}{\partial y}[y]$$

$$= x^2+x^3$$

Ex: $\frac{\partial}{\partial z}\left[xyz + z^2 e^x \sin(y)\right]$

$= xy \frac{\partial}{\partial z}[z] + e^x \sin(y) \frac{\partial}{\partial z}[z^2]$

$= xy + 2ze^x \sin(y)$

$f(x,y)$ NEVER "$\cancel{f'}$"

$\frac{\partial f}{\partial x} \equiv f_x$ $\frac{\partial f}{\partial y} \equiv f_y$

$$\begin{tikzcd} f \arrow[d, "\frac{\partial}{\partial x}"] \\ f_x \arrow[d, "\frac{\partial}{\partial y}"] \\ f_{xy} \end{tikzcd}$$

$$\frac{\partial}{\partial y}\frac{\partial}{\partial x} f = \boxed{\frac{\partial^2 f}{\partial y \partial x} \equiv f_{xy}}$$

$$\frac{\partial^5 f}{\partial^2 y \partial x \partial y \partial x} = \underset{\substack{\uparrow \\ 5^{th}}}{\frac{\partial}{\partial y}} \; \frac{\partial}{\partial y} \; \frac{\partial}{\partial x} \; \underset{\substack{\uparrow \\ 2^{nd}}}{\frac{\partial}{\partial y}} \; \underset{\substack{\uparrow \\ 1^{st}}}{\frac{\partial}{\partial x}} f$$