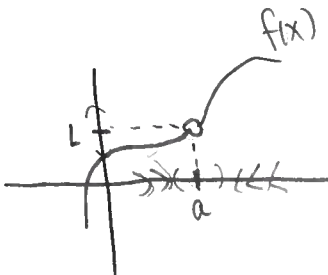
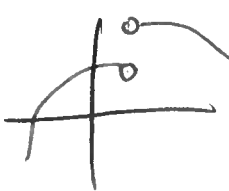



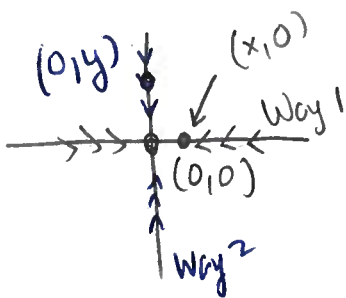
Calc 1:  $\lim_{x \rightarrow a} f(x) = L$  (1)

CALC 3: $z = f(x, y)$  $\lim_{(x, y) \rightarrow (a, b)} f(x, y) = L$

Ex: If $f(x, y) = x^2 y^3 + xy + 1$
 then $\lim_{(x, y) \rightarrow (1, 2)} f(x, y) = (1^2)(2^3) + (1)(2) + 1 = 11$



Ex: Show that

$\lim_{(x, y) \rightarrow (0, 0)} \frac{x^2 - y}{x^2 + y^2}$ DNE



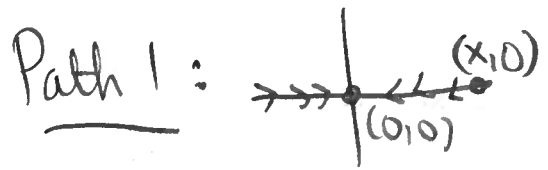
Way 1: $\lim_{(x, 0) \rightarrow (0, 0)} \frac{x^2 - 0}{x^2 + 0^2} = \lim_{x \rightarrow 0} \frac{x^2}{x^2} = 1$

Way 2: $\lim_{(0, y) \rightarrow (0, 0)} \frac{0 - y}{0 + y^2} = \lim_{y \rightarrow 0} \frac{-1}{y}$ DNE

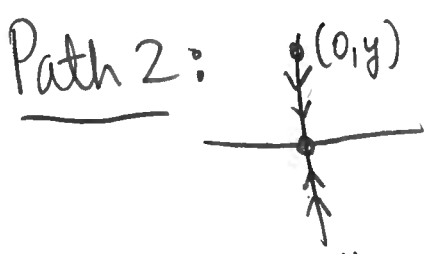



disagree \implies lim DNE

Ex: $f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}$ as $(x,y) \rightarrow (0,0)$



$$\lim_{(x,0) \rightarrow (0,0)} \frac{x^2 - 0^2}{x^2 + 0^2} = \lim_{x \rightarrow 0} 1 = 1$$

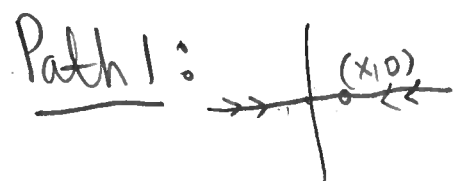


$$\lim_{(0,y) \rightarrow (0,0)} \frac{0 - y^2}{0 + y^2} = \lim_{y \rightarrow 0} -1 = -1$$

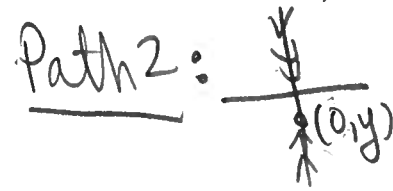
⇓

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2} \text{ DNE}$$

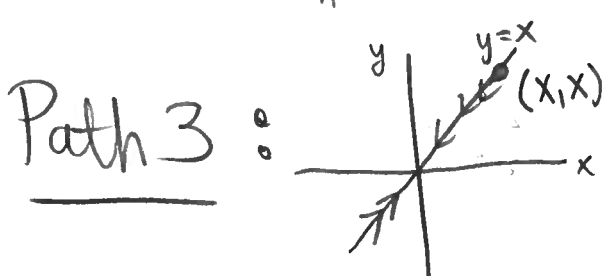
Ex: $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$



$$\lim_{(x,0) \rightarrow (0,0)} \frac{\overset{x \cdot 0}{0}}{x^2 + 0^2} = \lim_{x \rightarrow 0} 0 = 0$$



$$\lim_{(0,y) \rightarrow (0,0)} \frac{0 \cdot y}{0^2 + y^2} = \lim_{y \rightarrow 0} 0 = 0$$

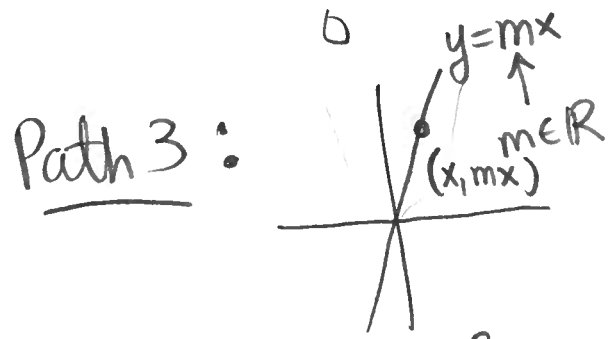
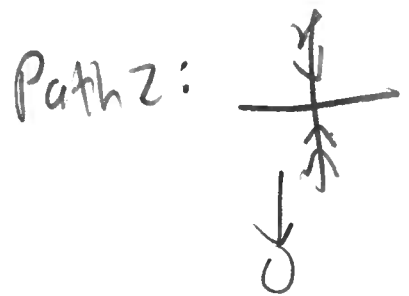
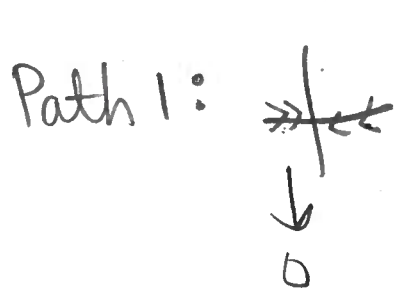


$$\lim_{(x,x) \rightarrow (0,0)} \frac{x^2}{2x^2} = \lim_{x \rightarrow 0} \frac{1}{2} = \frac{1}{2}$$

↑
y

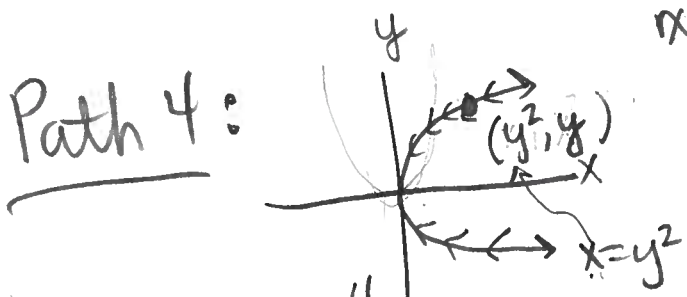
∴ original limit DNE

Ex: $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^2}$



$$\lim_{(x, mx) \rightarrow (0,0)} \frac{x(mx)^2}{x^2+(mx)^2} = \lim_{x \rightarrow 0} \frac{m^2 x^3}{x^2+m^2 x^2}$$

$$= \lim_{x \rightarrow 0} \frac{m^2 x}{1+m^2} = 0$$



$$\lim_{(y^2, y) \rightarrow (0,0)} \frac{y^4}{(y^2)^2+y^2} = \lim_{y \rightarrow 0} \frac{y^4}{y^4+y^2} \frac{(\frac{1}{y^4})}{(\frac{1}{y^4})}$$

L.H. = $\lim_{y \rightarrow 0} \frac{4y^3}{4y^3+2y}$

L.H. = $\lim_{y \rightarrow 0} \frac{12y^2}{12y^2+2} = 0$