# Calculus of space curves

✓ [calc on space curves] $f : \mathbb{R} \to \mathbb{R}^n$ $\quad \vec{r}(t) = \langle x(t), y(t) \rangle$

(coming soon) $f : \mathbb{R}^n \to \mathbb{R}$ $\quad z = f(x,y)$

(vector calc) $f : \mathbb{R}^n \to \mathbb{R}^m$

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~~vector~~ space curve

- $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$
- arithmetic works componentwise

Consequence:

$$\lim_{t \to a} \vec{r}(t) = \left\langle \lim_{t \to a} x(t), \lim_{t \to a} y(t), \lim_{t \to a} z(t) \right\rangle$$

$$\frac{d}{dt} \vec{r}(t) = \vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

$$\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b x(t)\,dt, \int_a^b y(t)\,dt, \int_a^b z(t)\,dt \right\rangle$$

Ex: Compute

$$\lim_{t \to 4} \left\langle \frac{1}{t}, t+7, \sin(t) \right\rangle$$

$$= \left\langle \frac{1}{4}, 11, \sin(4) \right\rangle$$

Ex: $$\lim_{t \to 1} \left\langle \frac{\sqrt{t}-1}{t^2+t-2}, \frac{t^8-1}{t-1}, 5 \right\rangle$$

$\uparrow \frac{0}{0}$ $\qquad$ $\uparrow \frac{0}{0}$

$$\frac{\sqrt{t}-1}{(t-1)(t+2)} = \left(\frac{(\sqrt{t}-1)}{(t-1)(t+2)}\right)\left(\frac{\sqrt{t}+1}{\sqrt{t}+1}\right) = \frac{(t-1)}{(t-1)(t+2)(\sqrt{t}+1)}$$

$$\lim_{t \to 1} \frac{t^8-1}{t-1} = \frac{0}{0} \overset{L.H.}{=} \lim_{t \to 1} \frac{8t^7}{1} = 8$$

$$= \left\langle \frac{1}{4}, 8, 5 \right\rangle$$

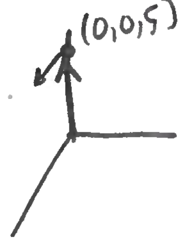
Ex: $\begin{cases} \vec{f}(t) = \langle 8t, 6t^3, 8t^4 + 5 \rangle \\ 0 \le t \le 1 \end{cases}$

↖ position

$\vec{f}'$ ~ velocity

$\vec{f}''$ ~ accel

Where does it start? $\vec{f}(0) = \langle 0, 0, 5 \rangle$




Where does it end? $\vec{f}(1) = \langle 8, 6, 13 \rangle$

Compute $\vec{f}'(t)$ $\quad \vec{f}'(t) = \langle 8, 18t^2, 32t^3 \rangle$

↖ vel.

Starting velocity? $\vec{f}'(0) = \langle 8, 0, 0 \rangle$

Ending velocity? $\vec{f}'(1) = \langle 8, 18, 32 \rangle$

Starting speed? $\|\vec{f}'(0)\| = 8$

Ending speed? $\|\vec{f}'(1)\| = \sqrt{8^2 + 18^2 + 32^2}$

$= 37.57$

↑

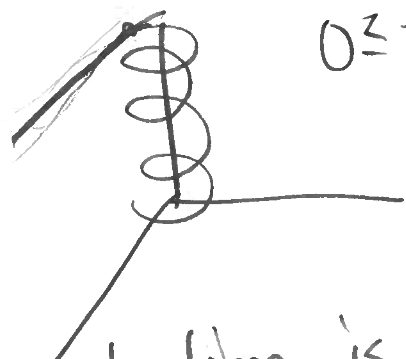
speed = ||velocity||

(4)

$\cos(\pi t)i + \sin(\pi t)j + 4tk$

Ex: $\vec{r}(t) = \langle \cos(\pi t), \sin(\pi t), \frac{t}{5} \rangle$ ← z-height

$0 \le t \le T$



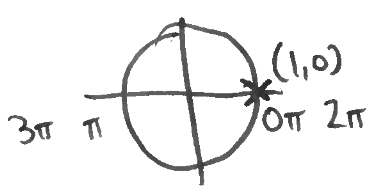
- at what time is particle at height 6?

$$\frac{t}{5} = 6 \implies \boxed{t = 30}$$

- What is velocity when particle is at height 6?

$$\vec{r}'(t) = \langle -\pi \sin(\pi t), \pi \cos(\pi t), \frac{1}{5} \rangle$$

$$\vec{r}'(30) = \langle -\pi \sin(30\pi), \pi \cos(30\pi), \frac{1}{5} \rangle$$




$$= \langle 0, \pi, \frac{1}{5} \rangle$$

- at height 6, particle moves in the tangent to velocity in (b). Find eqt of that line.

① parallel vec: $\vec{v} = \langle 0, \pi, \frac{1}{5} \rangle$

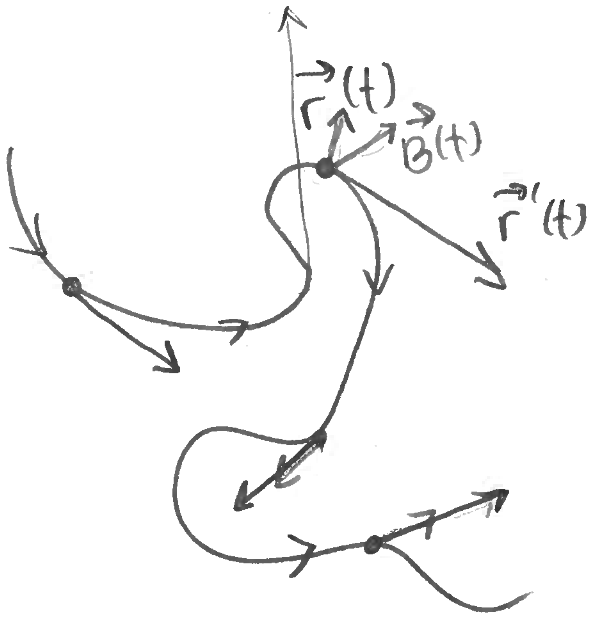
② point: $\vec{r}(30) = \langle 1, 0, 6 \rangle$

$$\vec{\ell}(t) = \langle 1, 0, 6 \rangle + t \langle 0, \pi, \frac{1}{5} \rangle$$

$$= \langle 1, \pi t, 6 + \frac{t}{5} \rangle$$

$$\vec{r}(30) = \langle \cos(30\pi), \sin(30\pi), \frac{30}{5} \rangle$$

$$= \langle 1, 0, 6 \rangle$$

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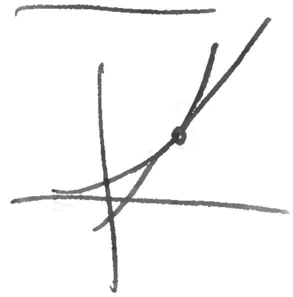
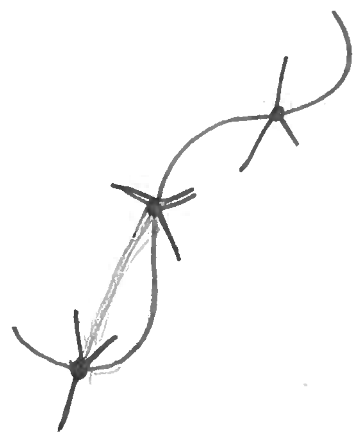


unit tangent vector

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$$



normal vector

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$$

binormal

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$