T Online HW now accessible to all (watch video)

Ex: $4x^2 - y^2 + 2z^2 + 4 = 0$

x-trace

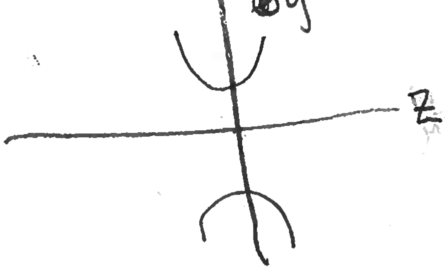
$x = k$

$4k^2 - y^2 + 2z^2 + 4 = 0$

$2z^2 - y^2 = -4 - 4k^2$

$\underbrace{y^2 - 2z^2 = 4 + 4k^2}_{\text{hyperbolas}}$

in yz-plane

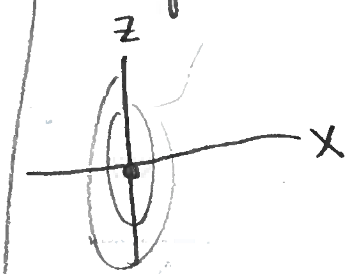


y-trace

$y = k$

$4x^2 + 2z^2 = \underbrace{-4 + k^2}_{|k| \geq 2}$

ellipses



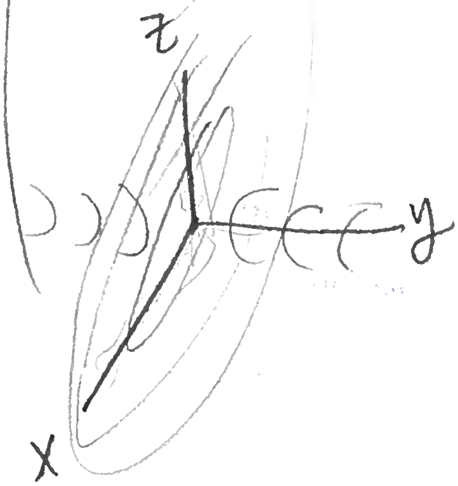
hyperboloid in 2 sheets

z-trace

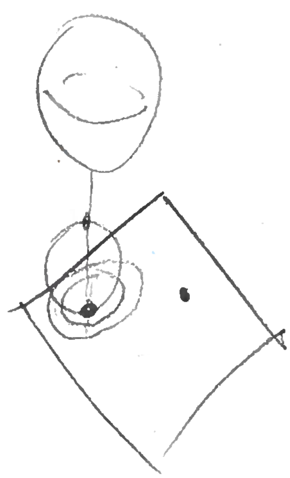
$z = k$

$4x^2 - y^2 = -4 - 2x^2$

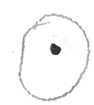
hyperbolas



Summary

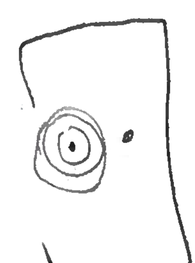


name	equation
ellipsoid	$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$
hyperboloid in 1-sheet	$-$
hyperboloid in 2 sheets	$-\quad -$
elliptic paraboloid	$z = x^2/a^2 + y^2/b^2$
hyperbolic paraboloid	$z = x^2/a^2 - y^2/b^2$



Level surfaces

"Flatland" ~ book from 1800's



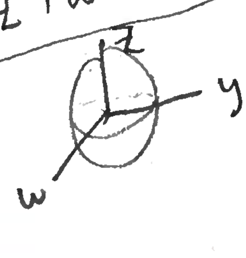
$x^2+y^2+z^2+w^2=1 \longrightarrow$ lives in 4D

x-traces, etc $\longrightarrow$ eq'ts of surfaces

$x=3$ (in $\mathbb{R}^4$)

$\Downarrow$

a copy of $\mathbb{R}^3$ along y, z, w axes $\Longrightarrow$ $\boxed{y^2+z^2+w^2=1-k^2}$ $|k|<1$



Parametrization of curves

- saw earlier for lines

$$\vec{r}(t) = \sim\sim\sim$$

$a \leq t \leq b$

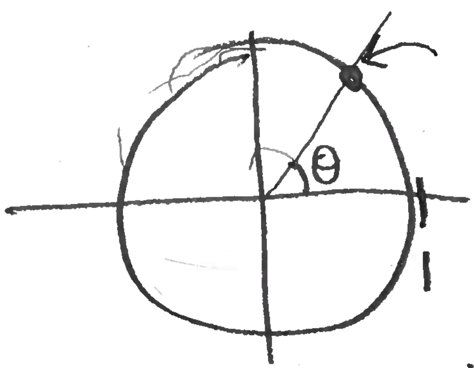
parametrization of a curve

- generally ~ think about

$$\vec{r}(t) = \langle x(t), y(t) \rangle \sim \text{in } \mathbb{R}^2$$

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle \sim \text{in } \mathbb{R}^3$$

- Recall unit circle

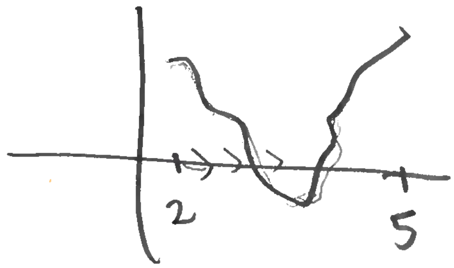


$(\cos\theta, \sin\theta)$

this is a parametrization of a circle of rad 1!

$$\begin{cases} \vec{r}(t) = \langle \cos t, \sin t \rangle \\ 0 \leq t \leq 2\pi \end{cases}$$

- Parametrize $y=f(x)$



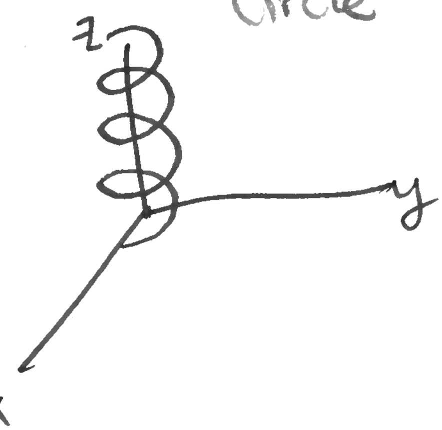
$$\begin{cases} \vec{r}(t) = \langle t, f(t) \rangle \\ 2 \le t \le 5 \end{cases}$$

- 3D parametrization

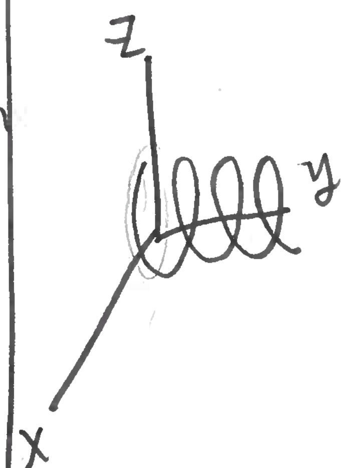
$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle$$

x y z

circle up&down

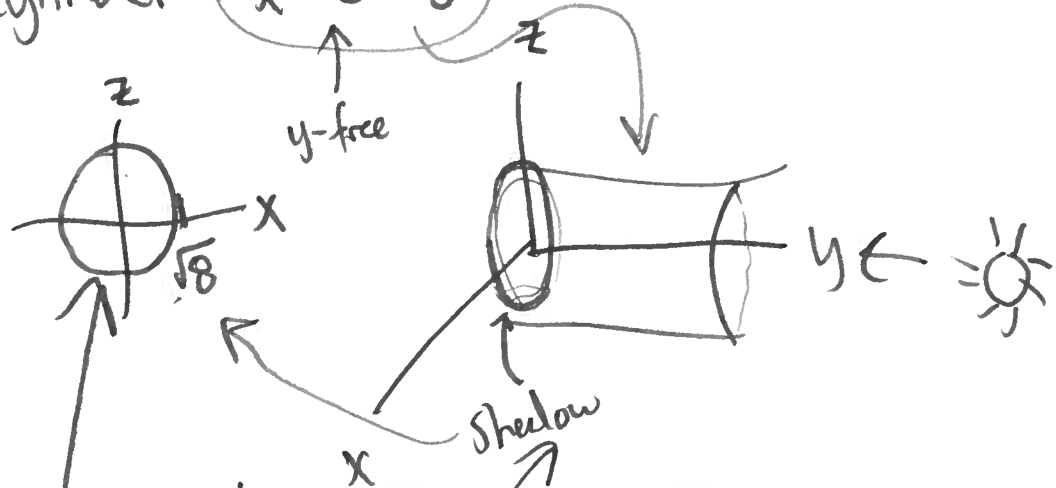


$\vec{r}(t) = \langle \cos t, t, \sin t \rangle$



① Find parametrization of the shadow → 2D of cylinder $x^2+z^2=8$ in the xz-plane.

Soln:



So, just parametrize this

$$\langle \sqrt{8}\cos t,\ 0,\ \sqrt{8}\sin t \rangle$$

$$0 \le t \le 2\pi$$

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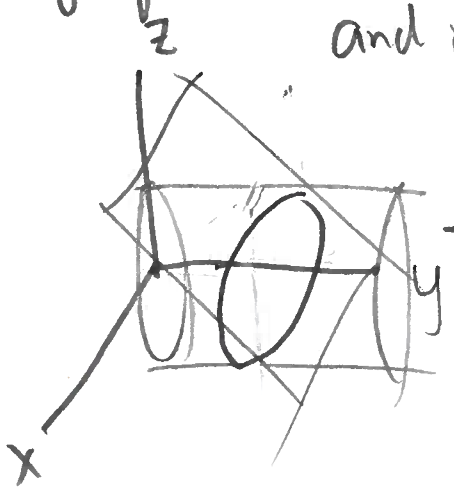
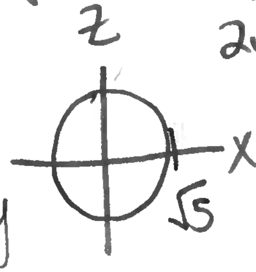
② eqt for the intersection of cylinder $x^2+z^2=5$ and plane $3x-2y+z=4$ → solve for y

$2y = 3x+z-4$





$$\vec{r}(t) = \langle \sqrt{5}\cos t,\ 3\sqrt{5}\cos t + \sqrt{5}\sin t - 4,\ \sqrt{5}\sin t \rangle$$