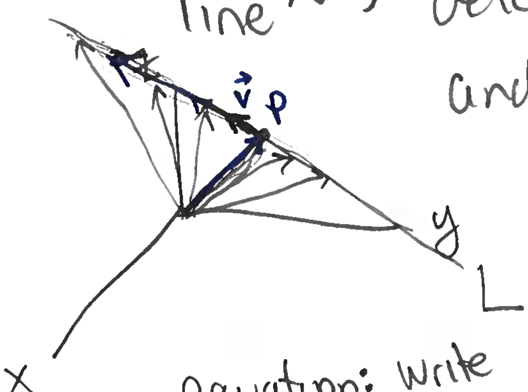
Written HW1 → Blackboard

"due" ~~today~~ → 11:59PM

Online HW → "due" by ~~Wed~~

Equations + lines + planes in space

line ⇝ determined by a point and a vector parallel to line



point $\qquad$ vector $\parallel L$

equation: write $P=(p_1,p_2,p_3)$, $\vec{v}=\langle v_1,v_2,v_3\rangle$

$\vec{p}=\langle p_1,p_2,p_3\rangle$

$$\boxed{\vec{r}(t)=\vec{p}+t\vec{v}}$$

$$\boxed{t\in\mathbb{R}}$$

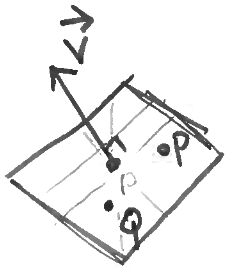
vector function

vector-valued function

Ex: Find eqt of line thru $(1,2,3)$ parallel to $\vec{v}=\langle 8,1,-6\rangle$ $\quad P$

$$\vec{r}(t)=\langle 1,2,3\rangle+t\langle 8,1,-6\rangle=\langle \underset{\text{"x"}}{1+8t},\ \underset{\text{"y"}}{2+t},\ \underset{\text{"z"}}{3-6t}\rangle$$

Eqt of planes



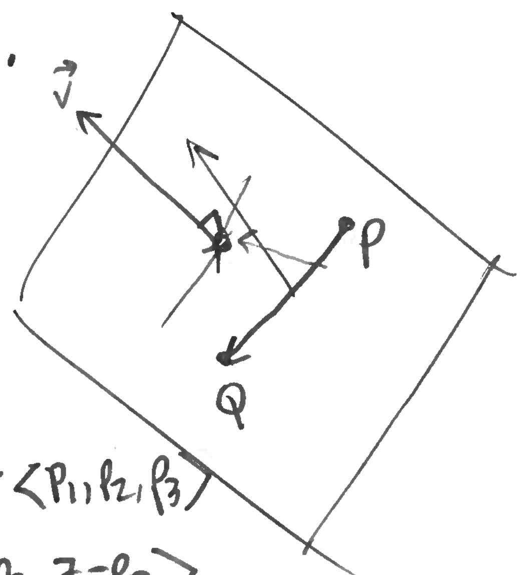
determined by a point on plane and a vector orthogonal to plane

<u>equation</u>: point $P = (p_1, p_2, p_3)$

vector $\vec{v} = \langle v_1, v_2, v_3 \rangle$ (orthog)

think about arbitrary point $Q = (x, y, z) \leftarrow$ on plane

We know $\vec{v}$ is orthogonal to all vectors in this plane.



(Hp-tail)

$$\vec{PQ} = \langle x, y, z \rangle - \langle p_1, p_2, p_3 \rangle$$
$$= \langle x - p_1, y - p_2, z - p_3 \rangle$$

<u>We know:</u> $\vec{v} \cdot \vec{PQ} = 0$

$$\langle v_1, v_2, v_3 \rangle \cdot \langle x - p_1, y - p_2, z - p_3 \rangle = 0$$

$$v_1(x - p_1) + v_2(y - p_2) + v_3(z - p_3) = 0$$

General plane: $\boxed{ax + by + cz = d}$

①

EX: Find eqt of plane ~~thru~~ containing $(2,7,1)$
w/ orthogonal vector $\vec{v} = \vec{i} = \langle 1,0,0\rangle$

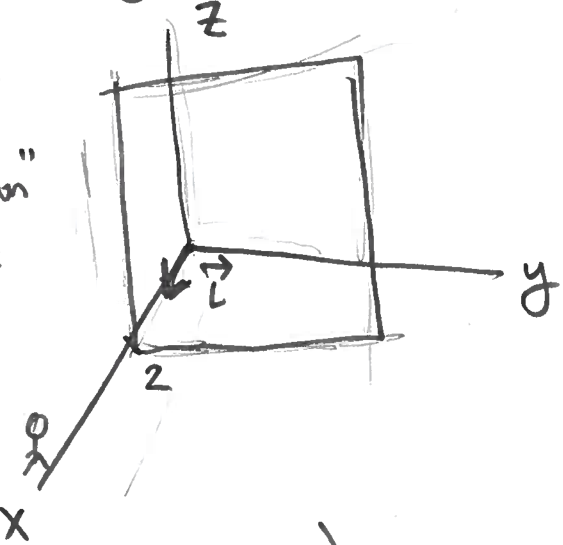
Soln:

$$\langle 1,0,0\rangle \cdot \langle x-2,\ y-7,\ z-1\rangle = 0$$

$$x-2+0+0=0$$

$$x=2$$

"no restriction" on y and z



EX: eqt of plane thru $(2,5,-6)$
w/ orthog vector $\vec{v}=\langle -1,2,8\rangle$

$$\langle -1,2,8\rangle \cdot \langle x-2,\ y-5,\ z+6\rangle = 0$$

$$-(x-2)+\overset{2y-10}{2(y-5)}+8(z+6)=0$$

solve for z

$$8z = x-2-2y+10-48 \longrightarrow 8z = x-2y-40$$

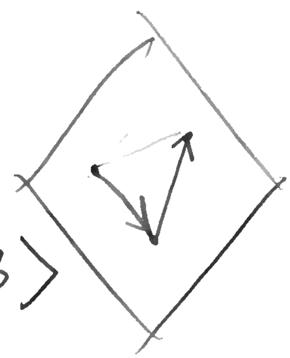
$$-x+2+2y-10+8z+48=0 \longrightarrow z = \frac{x}{8}-\frac{y}{4}-5$$

$$-x+2y+8z+40=0$$

2 points determine a line

3 points determine a plane
(noncollinear)

EX: Find eqt of plane containing A= $(1,1,6)$, B= $(2,-1,3)$ and C= $(5,1,-1)$.



$\vec{AB} = \langle 2,-1,3\rangle - \langle 1,1,6\rangle = \langle 1,-2,-3\rangle$

$\vec{BC} = \langle 5,1,-1\rangle - \langle 2,-1,3\rangle = \langle 3,2,-4\rangle$

$\vec{v} = \vec{AB} \times \vec{BC} = \langle 1,-2,-3\rangle \times \langle 3,2,-4\rangle$

$$= \det\left(\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -3 \\ 3 & 2 & -4 \end{bmatrix}\right)$$

$$= \vec{i}\,(8-(-6)) - \vec{j}\,(-4-(-9)) + \vec{k}\,(2-(-6))$$

$$= \langle 14,-5,8\rangle$$

$\langle 14,-5,8\rangle \cdot \langle x-1, y-1, z-6\rangle = 0$

$14(x-1) - 5(y-1) + 8(z-6) = 0$

$14x - 5y + 8z = 14 - 5 + 48 = 57$

43
14