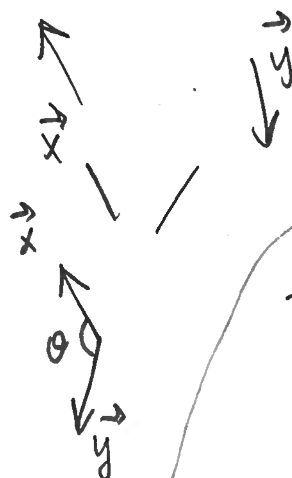
Dot Product $\vec{x} = \langle x_1, \dots, x_n \rangle$ $\vec{y} = \langle y_1, \dots, y_n \rangle$

$$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

$$\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

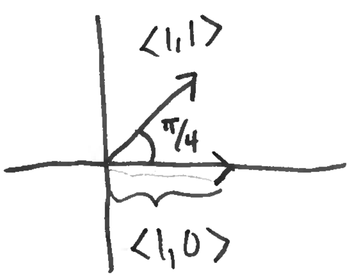
Dot product tells us about the angle between two vectors:

Theorem: $\vec{x} \cdot \vec{y} = \|\vec{x}\| \, \|\vec{y}\| \cos(\theta)$,
where θ is angle between them



Ex: Find $\langle 1,1 \rangle \cdot \langle 1,0 \rangle$ two ways

Soln:



① $\langle 1,1 \rangle \cdot \langle 1,0 \rangle = 1 \cdot 1 + 1 \cdot 0 = 1 + 0 = 1$

② $\langle 1,1 \rangle \cdot \langle 1,0 \rangle = \|\langle 1,1 \rangle\| \, \|\langle 1,0 \rangle\| \cos\left(\frac{\pi}{4}\right)$

$$= (\sqrt{2})(1)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{2}{2} = 1$$

TRIG

$2\pi \text{ rad} = 360^\circ$

$1 \text{ rad} = \left(\frac{360}{2\pi}\right)^\circ$

$x \text{ rad} = x\left(\frac{360}{2\pi}\right)^\circ$

CALC I

x in rad

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \sin\left(\frac{360}{2\pi} x\right) = \frac{360}{2\pi} \cos(\text{—})$

$\frac{d}{dx} \sin(x^\circ) = \frac{180}{\pi} \cos(x^\circ)$

$\cos^{-1}$ ②

Corollary: $\theta = \arccos\left(\dfrac{\vec{x}\cdot\vec{y}}{\|\vec{x}\|\,\|\vec{y}\|}\right)$

Ex: Find angle between $\langle 1,1\rangle$ and $\langle 1,0\rangle$

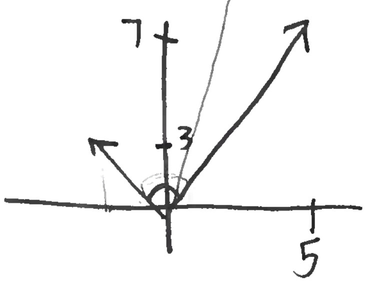
$$\theta = \arccos\left(\frac{\langle 1,1\rangle\cdot\langle 1,0\rangle}{\|\langle 1,1\rangle\|\,\|\langle 1,0\rangle\|}\right)$$

$$= \arccos\left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$= \frac{\pi}{4}$$

Ex: Find angle b/w $\langle -2,3\rangle$ and $\langle 5,7\rangle$



$$\theta = \arccos\left(\frac{-10+21}{\sqrt{4+9}\,\sqrt{25+49}}\right)$$

$$\begin{array}{r} 49 \\ 25 \\ \hline 74 \end{array}$$

$$= \arccos\left(\frac{11}{\sqrt{13}\,\sqrt{74}}\right)$$

$$\approx 1.21 \text{ rad} = (1.21 \text{ rad})(1)$$

$2\pi \text{ rad} = 360^\circ$

$1 = \dfrac{360^\circ}{2\pi \text{ rad}} = \dfrac{180^\circ}{\pi \text{ rad}}$

$$= (1.21 \text{ rad})\left(\frac{180^\circ}{\pi \text{ rad}}\right)$$

$$\approx 69.33^\circ$$

Ex: Find angle b/w

⟨1,2,3,4⟩ and ⟨-1,0,1,1⟩

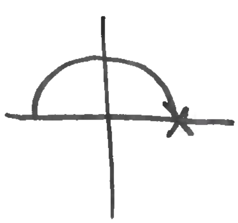
$$\theta = \arccos\left(\frac{-1+0+3+4}{\sqrt{1+4+9+16}\ \sqrt{1+0+1+1}}\right)$$

$$= \arccos\left(\frac{6}{\sqrt{30}\ \sqrt{3}}\right) \approx 0.89 \text{ rad}$$

Ex: Find angle b/w

⟨1,1⟩ and ⟨2,2⟩

$$\theta = \arccos\left(\frac{2+2}{\sqrt{2}\ \sqrt{8}}\right) = \arccos\left(\frac{4}{\sqrt{16}}\right)$$

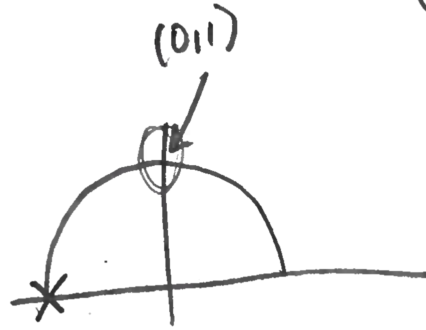


$$= \arccos(1)$$

$$= 0$$

<u>Ex</u>: Angle b/w

$\langle 1,1\rangle$ and $\langle -1,-1\rangle$



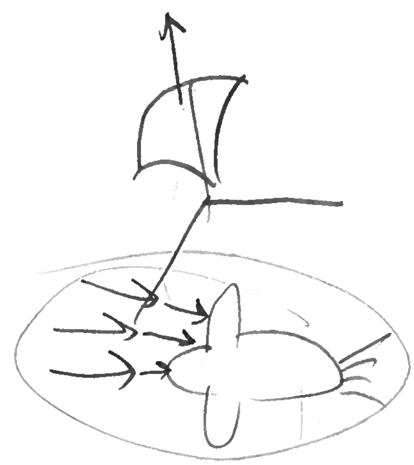
$$\theta = \arccos\left(\frac{-2}{\sqrt{2}\sqrt{2}}\right)$$

$$= \arccos\left(\frac{-2}{2}\right) = \arccos(-1) = \pi$$

<u>FACT</u>: perpendicular vectors have a dot

$\longrightarrow$ ("orthogonal")

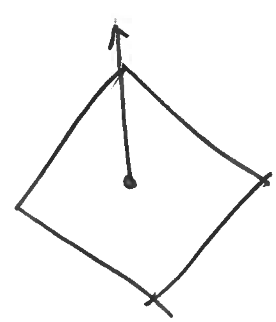
product of 0



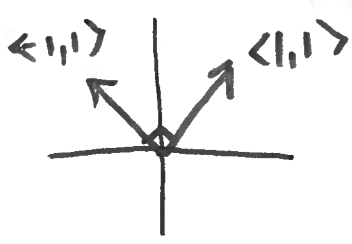
<u>Ex</u>: $\vec{i} \cdot \vec{j} = \langle 1,0\rangle \cdot \langle 0,1\rangle$

$= 0+0 = 0$

<u>Ex</u>: Show $\langle 1,-1\rangle$ and $\langle -1,1\rangle$ are orthogonal.



<u>Soln</u>: $\langle 1,1\rangle \cdot \langle -1,1\rangle = -1+1 = 0$



So far

vec + vec = vec

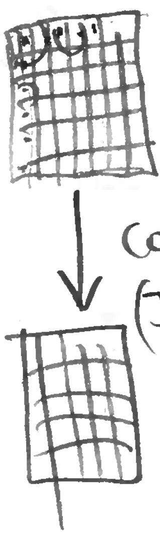
(number) vec = vect

vec • vec = number

"natural way"

$\langle 1, 2, 3 \rangle \otimes \langle 2, 1, 6 \rangle = \langle 2, 2, 18 \rangle$

"Hadamard product"



compress (JPEG)

NOT HERE

- Cross product — way to generate a vector perpendicular to two given vectors.

- FOR US — ONLY works in $\mathbb{R}^3$

Aside ~ matrices ← grid of numbers

$\begin{bmatrix}1&2\\1&5\end{bmatrix}$ $\quad \underset{\text{rows}}{2}\times\underset{\text{cols}}{2}$

$\cancel{\begin{bmatrix}1&2\\1&1\\6&5\end{bmatrix}}$ $\quad 3\times2$

$\begin{bmatrix}1&2&3\\3&4&5\\1&1&6\end{bmatrix}$ $\quad 3\times3$

square

Determinant

$$\det\left(\begin{bmatrix}a&b\\c&d\end{bmatrix}\right) = ad - bc$$

Ex: $\det\left(\begin{bmatrix}1&2\\3&4\end{bmatrix}\right) = 1\cdot4 - 2\cdot3$

$= 4 - 6$

$= -2$