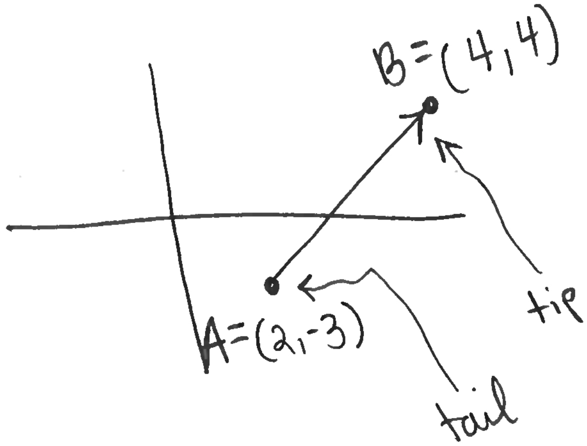
Vectors



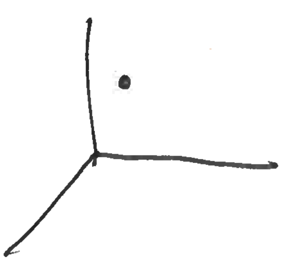
To take points & turn into a vector:
"tip − tail"

$$\overrightarrow{AB} = \text{"}B - A\text{"} = \langle 4,4\rangle - \langle 2,-3\rangle$$
$$= \langle 2, 7\rangle$$

We say $\vec{a}$ is parallel to $\vec{b}$ whenever there is a constant c s.t.
$\vec{a} = c\vec{b}$

ex) $\langle 1,1\rangle$ and $\langle 3,3\rangle$ are parallel
b/c
$\langle 3,3\rangle = 3\langle 1,1\rangle$

Ex) $\langle 1,5\rangle$ is not parallel to $\langle -1,-1\rangle$
because:



$$\langle 1,5\rangle = c\langle -1,-1\rangle \longrightarrow \begin{cases} 1 = -c \\ 5 = -c \end{cases}$$
$$= \langle -c,-c\rangle$$
$$\rightarrow \begin{cases} c = -1 \\ c = -5 \end{cases} \ast$$

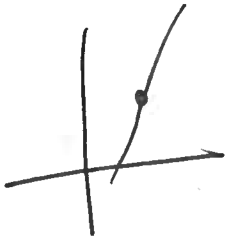
$\times$

in 2D

point, slope

$\Downarrow$

eqt of line

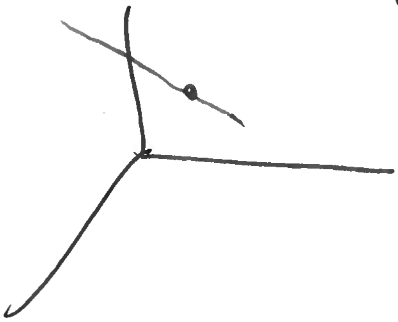


in 3D

point, vector parallel to line

$\Downarrow$

eqt of line



Ex: If $A=(1,1)$ and $\vec{AB} = \langle 3,3\rangle$

what is B?

Soln: $\vec{AB} =$ "$B - A$"

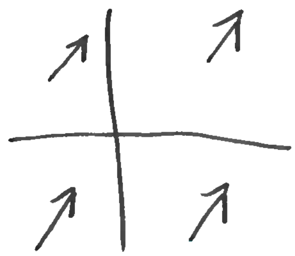
$$\langle 3,3\rangle = \vec{AB} = \langle b_1, b_2\rangle - \langle 1,1\rangle$$

$$\langle 3,3\rangle = \langle b_1, b_2\rangle - \langle 1,1\rangle$$

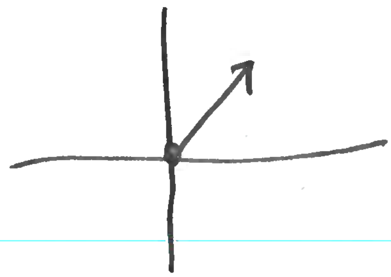
add $\langle 1,1\rangle$

$$\Rightarrow \langle 4,4\rangle = \langle b_1, b_2\rangle$$

Standard position



put tail at $(0,0)$



Ex: For which t does

$$\langle \underbrace{t+5}_{x}, \underbrace{t-1}_{y} \rangle = \langle 8, 2 \rangle \text{ hold?}$$

Soln:

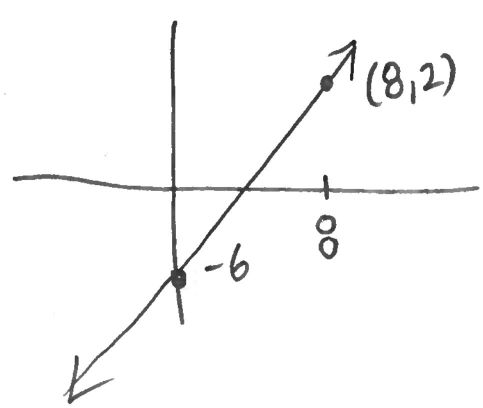
$$\begin{cases} t+5=8 \longrightarrow t=3 \ \checkmark \\ t-1=2 \longrightarrow t=3 \end{cases}$$

$$\begin{cases} x=t+5 \\ y=t-1 \longrightarrow y+1=t \end{cases}$$

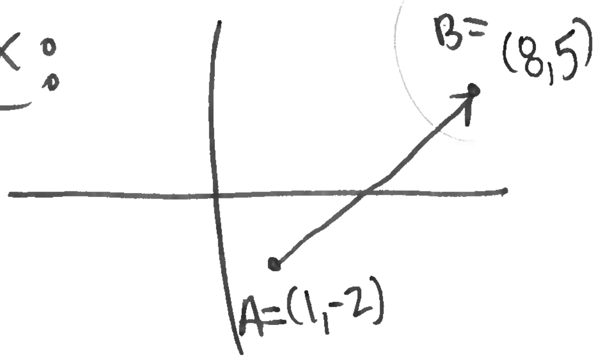
$x=(y+1)+5$

$x=y+6$

$y=x-6$



EX:



① Find $\overrightarrow{AB}$

② Write $\overrightarrow{AB}$ as a sum of two nonzero nonparallel vectors.

Soln: ① tip - tail $= \langle 8,5 \rangle - \langle 1,-2 \rangle$

$= \langle 7,7 \rangle$

② $\langle 7,0 \rangle + \langle 0,7 \rangle$

$\langle 1,6 \rangle + \langle 6,1 \rangle$

etc...

Dot Products, Magnitude, + Direction

- given $\vec{x} = \langle x_1, x_2, \dots, x_n \rangle$
  $\vec{y} = \langle y_1, y_2, \dots, y_n \rangle$

$\left(\begin{array}{l} \vec{x} \in \mathbb{R}^n \\ \vec{x} \in \mathbb{R}^{n \times 1} \end{array} \right)$

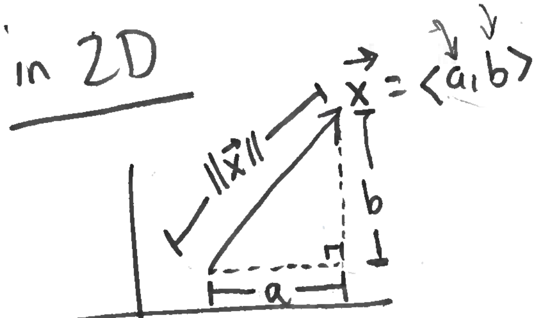
Dot product:

$$\underset{\substack{\uparrow \\ \text{vector}}}{\vec{x}} \cdot \underset{\substack{\uparrow \\ \text{vector}}}{\vec{y}} = \underbrace{x_1 y_1 + x_2 y_2 + \dots + x_n y_n}_{\text{number}}$$

- magnitude ~ ("length") ("norm")

$\|\vec{x}\|$

in 2D



$$a^2 + b^2 = \|\vec{x}\|^2$$

$$\|\vec{x}\| = \sqrt{a^2 + b^2}$$

in general

if $\vec{x} = \langle x_1, x_2, \dots, x_n \rangle$, then

$$\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

Relationship b/w dot prod + norm:

if $\vec{x} = \langle x_1, x_2, \dots, x_n \rangle$

$$\sqrt{\vec{x} \cdot \vec{x}} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2} = \|\vec{x}\|$$

$$\boxed{\vec{x} \cdot \vec{x} = \|\vec{x}\|^2}$$

Unit vectors - means norm 1

$\vec{u} = \langle u_1, u_2 \rangle$ $\quad \|\vec{u}\| = 1$

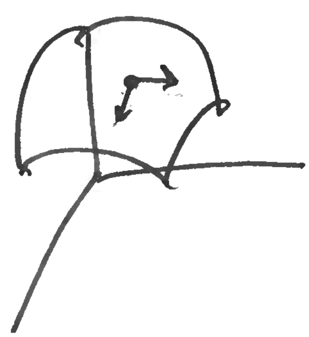
we say $\vec{u}$ is a unit vector

Ex: $\|\vec{i}\| = \|\langle 1, 0 \rangle\| = \sqrt{1^2 + 0^2} = 1$ unit

$\|\vec{j}\| = \|\langle 0, 1 \rangle\| = \sqrt{0^2 + 1^2} = 1$

$\|\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle\| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

$\|\langle 1, 1 \rangle\| = \sqrt{1^2 + 1^2} = \sqrt{2}$ not unit



Find a unit vector parallel to any nonzero vector

$\|\vec{x}\| \neq 0$ $\qquad$ $\vec{0} = \langle 0, 0, \dots, 0 \rangle$

<u>Define</u>: $\vec{w} = \frac{1}{\|\vec{x}\|}\vec{x}$

unit vector AND parallel to $\vec{x}$

$$\vec{x} = \langle x_1, \dots, x_n \rangle$$

$$\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

$$\|\vec{w}\| = \left\| \left\langle \frac{x_1}{\|\vec{x}\|}, \frac{x_2}{\|\vec{x}\|}, \dots, \frac{x_n}{\|\vec{x}\|} \right\rangle \right\|$$

$$= \sqrt{\left(\frac{x_1}{\|\vec{x}\|}\right)^2 + \dots + \left(\frac{x_n}{\|\vec{x}\|}\right)^2}$$

$$= \sqrt{\frac{x_1^2}{x_1^2 + \dots + x_n^2} + \dots + \frac{x_n^2}{x_1^2 + \dots + x_n^2}}$$

$$= \sqrt{\frac{x_1^2 + \dots + x_n^2}{x_1^2 + \dots + x_n^2}} = \sqrt{1} = 1$$

"normalization"

Find a unit vector $\vec{u}$ parallel to

$$\langle 3,5 \rangle$$

<u>Soln</u>: $\|\langle 3,5\rangle\| = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

$$3\vec{u} = \frac{1}{\sqrt{34}}\langle 3,5\rangle = \left\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{35}} \right\rangle$$

$$3\vec{u} = \left\langle \frac{9}{\sqrt{34}}, \frac{15}{\sqrt{34}} \right\rangle$$

<u>length 3!</u>