Written HW9 – MATH 2501 Fall 2020

Due by 30 September for timely completion credit

Full credit on these problems require a fully symbolic derivation of the solutions.

1. The Stefan–Boltzmann law is an equation that relates the power radiated from a black body (e.g. a black hole) in terms of its temperature. It is given by
$$J = \sigma T^4,$$
where J represents the total energy radiated per unit surface area of the black body per unit time, $\sigma \approx 5.670$ (in SI units) is a constant called the Stefan-Boltzmann constant, and T represents temperature (measured in Kelvin, K).

   If the temperature is decreasing at a rate of $3\frac{K}{\text{hr}}$, then how is the total energy radiated per unit surface area per unit time changing when the temperature is $T = 100K$?

2. In chemistry, "pH" is used to quantify the acidity of a solution. It is defined by the equation
$$pH = -\frac{1}{\ln(10)}\ln(a),$$
where a denotes the hydrogen ion activity measured in the solution.

   If the pH of the solution is increasing at a rate of $\frac{0.003}{\text{min}}$, then what is the rate of change of hydrogen ion activity when $pH = 3$? (*note: it appears that both pH and "hydrogen ion activity" are "unitless" quantities – this happens sometimes!*)

3. Joule's law says
$$P = I^2R,$$
where P is power (in watts), I is the current (in amps), and R is the resistance (in ohms, Ω).

   If the current is increasing at a rate of $2\frac{\text{amps}}{\text{sec}}$ and power is decreasing at a rate of $3\frac{\text{watts}}{\text{sec}}$, then how is resistance changing when the current is 12 amps and the power is 15 watts?