Written HW3 – MATH 2501 Fall 2020

Due by Thursday, 3 September for timely completion credit

An important theorem for limits is called the "squeeze theorem":

Squeeze theorem: If for all x near a, the inequality $g(x) \le f(x) \le h(x)$ holds and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = L$.

This theorem is useful when f is a complicated function, but you can find a g and h that are not so complicated. For full credit, clearly show how you are verifying the two conditions of the squeeze theorem and state "by the squeeze theorem" before you arrive at your conclusion.

1. Suppose you knew that for x near 1,

$$-4x \le f(x) \le x^2 - 6x + 1.$$

   Use the squeeze theorem to determine $\lim_{x\to 1} f(x)$.

2. Recall that $-1 \le \sin(x) \le 1$. Use the squeeze theorem to evaluate $\lim_{x\to 0} x^2 \sin\left(\frac{6}{x}\right)$.

3. Use the squeeze theorem to calculate

$$\lim_{x\to 0} \sqrt{|x|}e^{\sin\left(\frac{\pi}{x}\right)}.$$

   (*hint:* $e^{-1} \le e^{\sin\left(\frac{\pi}{x}\right)} \le e$)

4. Use the squeeze theorem to calculuate

$$\lim_{x\to 0} x \sin\left(\cos\left(\frac{1}{x}\right)\right)$$