Written HW17 – MATH 2501 Fall 2020

Due by 25 November for timely completion credit

In class, we discussed the famous "gamma function" which is defined for $x > 0$ by

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\mathrm{d}t.$$

We calculatd some of the values of this function in class using the fundamental theorem of calculus and a limit. We saw that it obeys the so-called recurrence formula

$$x\Gamma(x) = \Gamma(x+1),$$

and we discussed how

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

(3) The integral defining $\Gamma(x)$ shows that the area under a certain curve relates to the value of the gamma function. Use Desmos, as I did in class, to visualize the curve and the area beneath it for the values of $x = 3$ and $x = 4.5$. Your submission for this problem should include graphs that look similar to those I made in the 18 November lecture (around 10 minutes into the lecture video).

(3) Use the recurrence relation to determine the value of $\Gamma\left(\frac{5}{2}\right)$ in terms of the constant $\sqrt{\pi}$.

(3) Algebraically recurrence relation to solve it for $\Gamma(x)$.

(3) The formula you derived in the previous part allows the extension of values of the gamma function to the negative real line (*the integral actually doesn't work to define it there!*). Use the formula you obtained in the previous part to find a value of $\Gamma\left(-\frac{1}{2}\right)$ in terms of $\sqrt{\pi}$.

(3) What does the formula you derived in part (3) suggest about $\Gamma(0)$?